8.5 Area Between Curves (with respect to y)

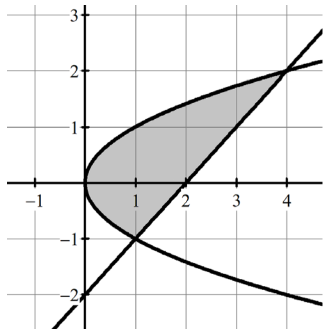
Calculus

Solutions

Practice

For each region, set up an integral **with respect to y** that represents the area of the region. Do not solve.

1. $x = y^2,\ x = y + 2$



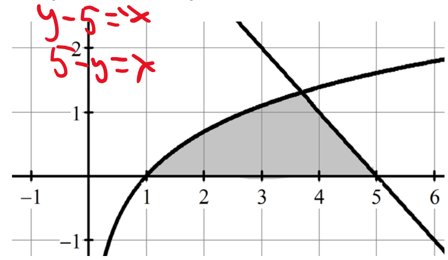
$$\boxed{\int_{-1}^{2}(y+2-y^2)\,dy}$$

2. $y = \ln x,\ y = 5 - x,\ y = 0$

$e^y = x$

$y-5=-x$

$5-y=x$



$$\boxed{\int_{0}^{1.3065}(5-y-e^y)\,dy}$$

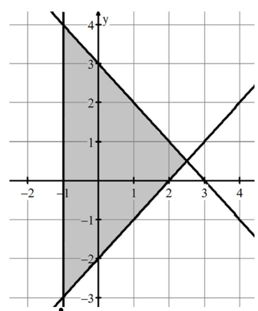
3. $y = -x + 3,\ y = x - 2$, and $x = -1$

$x=-y+3 \qquad x=y+2$

$-y+3=y+2$

$-2y=-1$

$y=\frac{1}{2}$



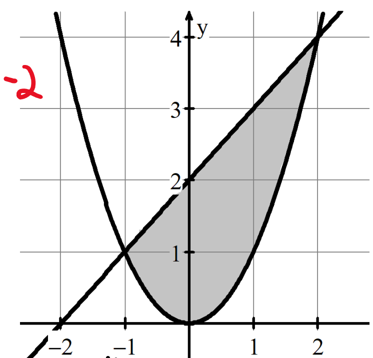
$$\int_{-3}^{\frac{1}{2}}(y+2--1)\,dy+\int_{\frac{1}{2}}^{4}(-y+3--1)\,dy$$

$$\boxed{\int_{-3}^{\frac{1}{2}}(y+3)\,dy+\int_{\frac{1}{2}}^{4}(4-y)\,dy}$$

4. $y = x^2,\ y = x + 2$

$x=\sqrt{y} \qquad x=y-2$

$x=-\sqrt{y}$



$$\int_{0}^{1}(\sqrt{y}--\sqrt{y})\,dy+\int_{1}^{4}(\sqrt{y}-(y-2))\,dy$$

$$\boxed{\int_{0}^{1}2\sqrt{y}\,dy+\int_{1}^{4}(\sqrt{y}-y+2)\,dy}$$

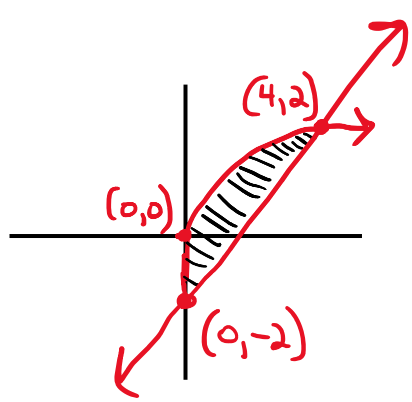
Set up the integral(s) that give the area of the region bounded by the given equations. Show the equivalent set up with respect to x as well as with respect to y.

5. $y = \sqrt{x}, x = 0$ and $y = x - 2$

Sketch a graph here in the middle!

with respect to x

$$\boxed{\int_{0}^{4}(\sqrt{x}-x+2)\,dx}$$



with respect to y

$x=y^2 \qquad x=y+2$

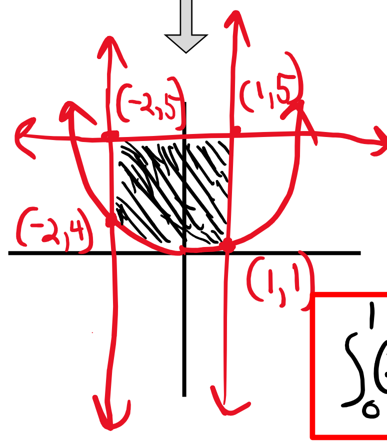
$$\boxed{\int_{-2}^{0}(y+2)\,dy+\int_{0}^{2}(y+2-y^2)\,dy}$$

6. $y = x^2, y = 5, x = -2, x = 1$

with respect to x

$$\boxed{\int_{-2}^{1} (5 - x^2)\, dx}$$

Sketch a graph here in the middle!



with respect to y

$x = \pm\sqrt{y}$

$$\int_0^1 (\sqrt{y} - -\sqrt{y})\, dy + \int_1^4 (1 - -\sqrt{y})\, dy + \int_4^5 (1 - -2)\, dy$$

$$\boxed{\int_0^1 (2\sqrt{y})\, dy + \int_1^4 (1 + \sqrt{y})\, dy + \int_4^5 3\, dy}$$

Find the area of the region bounded by the following curves. Set up your integrals with respect to y. A calculator is allowed to evaluate the integral.

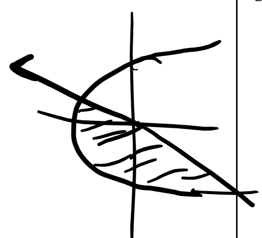
7. $x = y^2 - 4, \; x = -3y$

$y^2 - 4 = -3y$

$y^2 + 3y - 4 = 0$

$(y+4)(y-1) = 0$

$y = -4 \quad y = 1$



$$\boxed{\int_{-4}^{1} (-3y - y^2 + 4)\, dy = 20.833}$$

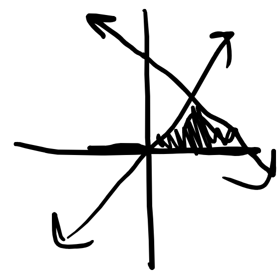
8. $y = x, \; y = 2 - x, \; y = 0$

$x = 2 - y$

$y = 2 - y$

$2y = 2$

$y = 1$



$$\int_0^1 (2 - y - y)\, dy$$

$$\boxed{\int_0^1 (2 - 2y)\, dy = 1}$$

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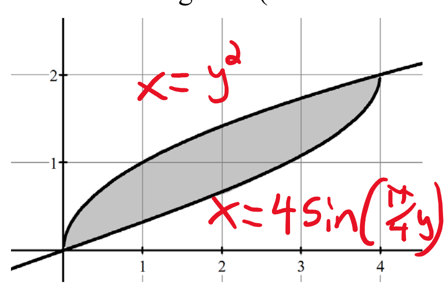
Test Prep

9. Solve the following WITHOUT the help of a calculator. Let R be the region bounded by the graphs of $y = \sqrt{x}$ on top and $y = \frac{4}{\pi}\sin^{-1}\left(\frac{x}{4}\right)$ and on bottom, as shown in the figure. What is the area of the region? (hint: integrating with respect to y is easier than with respect to x for this problem.)



$\frac{\pi}{4}y = \sin^{-1}\left(\frac{x}{4}\right)$

$\sin\left(\frac{\pi}{4}y\right) = \frac{x}{4}$

$4\sin\left(\frac{\pi}{4}y\right) = x$

u-sub $\quad y^2 = x$

$$\int_0^2 \left(4\sin\left(\frac{\pi}{4}y\right) - y^2\right) dy$$

$$\frac{4}{\pi} \cdot 4\left[-\cos\left(\frac{\pi}{4}y\right)\right] - \left[\frac{y^3}{3}\right] \Bigg|_0^2$$

$$-\frac{16}{\pi}\cos\left(\frac{\pi}{2}\right) - \frac{8}{3} \quad -\left(-\frac{16}{\pi}\cos(0) - 0\right)\Big]$$

$$0 - \frac{8}{3} - \left(-\frac{16}{\pi} - 0\right) = \boxed{-\frac{8}{3} + \frac{16}{\pi}}$$