Calculus

9.4 Derivatives of Vector-Valued Functions

Notes

Write your questions and thoughts here!

Vector basics:

- Vectors have magnitude (length) and direction.
- Vectors can be represented by directed line segments.
- Vectors are equal if they have the same direction and magnitude.
- Magnitude is designated by $\|v\|$
- Vectors have a horizontal and vertical component.
- Component form of a vector is $\langle x, y\rangle$

1. Find the component form and magnitude of the vector that has an initial point of $(1,2)$ and terminal point $(5,4)$.

   Component form:

   Magnitude:

Vector-Valued Functions: $r(t) = \langle f(t), g(t)\rangle$ where $f(t)$ and $g(t)$ are the component functions with the parameter t.

Differentiation of Vector-Valued Functions

If $r(t) = \langle f(t), g(t)\rangle$ then

Properties of the derivative for vector-valued functions

$$\frac{d}{dt}[c \cdot r(t)] = c \cdot r'(t)$$

$$\frac{d}{dt}[r(t) \cdot s(t)] = r'(t)s(t) + r(t)s'(t)$$

$$\frac{d}{dt}[r(t) \pm s(t)] = r'(t) \pm s'(t)$$

$$\frac{d}{dt}\left[r\big(s(t)\big)\right] = r'\big(s(t)\big) \cdot s'(t)$$

1. $r(t) = \langle 2t^2 + 4t + 1,\ 3t^3 - 4t\rangle$ then $r'(t) =$

2. $r(t) = \langle t^3 + 5,\ 2t\rangle$ find $\frac{d}{dt}r(2t)$

3. The path of a particle moving along a path in the xy-plane is given by the vector-valued function, $f(t) = \langle t^2, \sin t\rangle$. Find the slope of the path of the particle at $t = \frac{3\pi}{4}$.

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Practice

Each problem contains a vector-valued function. Find the given first or second derivative.

1. $f(t) = \langle 4t^3 + 2t^2 + 7t,\ 4t^2 + 3t \rangle$, then $f'(t) =$

2. $f(t) = \langle 3\sin 2t, 4\cos 3t \rangle$, then $f'\left(\frac{\pi}{6}\right) =$

3. $f(t) = \langle 3e^{2t},\ 5e^{4t} \rangle$, then $f''(t) =$

4. $f(t) = \langle t^{-2},\ (t+1)^{-1} \rangle$, then $f''(-2) =$

5. $f(t) = \langle e^t + e^{-t},\ e^t - e^{-t} \rangle$, then $f'(t) =$

6. $f(t) = \langle 2\sin 4t, 2\cos 3t \rangle$, then $f'(t) =$

7. $f(t) = \langle t\sin t,\ t\cos t \rangle$, then $f'\left(\frac{\pi}{2}\right) =$

8. $f(t) = \langle 3t^2 + 6t + 1,\ 4t^3 - 2t^2 + 6t \rangle$, then $f'(1) =$

9. The path of a particle moving along a path in the xy-plane is given by the vector-valued function, $f(t) = \langle t^3 + 2t^2 + t,\ 2t^3 - 4t \rangle$. Find the slope of the path of the particle at $t = 3$.

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10. The position of a particle moving in the xy-plane is defined by the vector-valued function, $f(t) = \langle t^3 - 6t^2,\ 2t^3 - 9t^2 - 24t \rangle$. For what value of $t \geq 0$ is the particle at rest?

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Test Prep

11. **Calculator active.** The path of a particle moving along a path in the xy-plane is given by the vector-valued function f and f' is defined by $f'(t) = \langle t^{-1},\ 2ke^{kt} \rangle$ where k is a positive constant. The line $y = 4x + 5$ is parallel to the line tangent to the path of the particle at the point where $t = 2$. What is the value of k?

12. At time t, $0 \leq t \leq 2\pi$, the position of a particle moving along a path in the xy-plane is given by the vector-valued function, $f(t) = \langle t \sin t, \cos 2t \rangle$. Find the slope of the path of the particle at time $t = \frac{\pi}{2}$.