

9.4 Derivatives of Vector-Valued Functions

Calculus

Solutions

Practice

Each problem contains a vector-valued function. Find the given first or second derivative.

1. $f(t) = \langle 4t^3 + 2t^2 + 7t, 4t^2 + 3t \rangle$, then $f'(t) =$

$$f'(t) = \langle 12t^2 + 4t + 7, 8t + 3 \rangle$$

2. $f(t) = \langle 3 \sin 2t, 4 \cos 3t \rangle$, then $f'(\frac{\pi}{6}) =$

$$f'(t) = \langle 6 \cos(2t), -12 \sin(3t) \rangle$$

$$f'(\frac{\pi}{6}) = \langle 6 \cos(\frac{\pi}{3}), -12 \sin(\frac{\pi}{2}) \rangle$$

$$= \langle 6(\frac{1}{2}), -12(1) \rangle$$

$$\langle 3, -12 \rangle$$

3. $f(t) = \langle 3e^{2t}, 5e^{4t} \rangle$, then $f''(t) =$

$$f'(t) = \langle 6e^{2t}, 20e^{4t} \rangle$$

$$f''(t) = \langle 12e^{2t}, 80e^{4t} \rangle$$

4. $f(t) = \langle t^{-2}, (t+1)^{-1} \rangle$, then $f''(-2) =$

$$f'(t) = \langle -2t^{-3}, -(t+1)^{-2} \rangle$$

$$f''(t) = \langle 6t^{-4}, 2(t+1)^{-3} \rangle$$

$$f''(-2) = \langle \frac{6}{(-2)^4}, \frac{2}{(-2+1)^{-3}} \rangle$$

$$\langle \frac{6}{16}, \frac{2}{-1} \rangle$$

$$\langle \frac{3}{8}, -2 \rangle$$

5. $f(t) = \langle e^t + e^{-t}, e^t - e^{-t} \rangle$, then $f'(t) =$

$$f'(t) = \langle e^t - e^{-t}, e^t + e^{-t} \rangle$$

6. $f(t) = \langle 2 \sin 4t, 2 \cos 3t \rangle$, then $f'(t) =$

$$f'(t) = \langle 8 \cos(4t), -6 \sin(3t) \rangle$$

7. $f(t) = \langle t \sin t, t \cos t \rangle$, then $f'(\frac{\pi}{2}) =$

$$f'(t) = \langle \sin t + t \cos t, \cos t - t \sin t \rangle$$

$$f'(\frac{\pi}{2}) = \langle \sin \frac{\pi}{2} + \frac{\pi}{2} \cos \frac{\pi}{2}, \cos \frac{\pi}{2} - \frac{\pi}{2} \sin \frac{\pi}{2} \rangle$$

$$= \langle 1 + \frac{\pi}{2}(0), 0 - \frac{\pi}{2}(1) \rangle$$

$$\langle 1, -\frac{\pi}{2} \rangle$$

8. $f(t) = \langle 3t^2 + 6t + 1, 4t^3 - 2t^2 + 6t \rangle$, then $f'(1) =$

$$f'(t) = \langle 6t + 6, 12t^2 - 4t + 6 \rangle$$

$$f'(1) = \langle 6 + 6, 12 - 4 + 6 \rangle$$

$$f'(1) = \langle 12, 14 \rangle$$

9. The path of a particle moving along a path in the xy -plane is given by the vector-valued function, $f(t) = \langle t^3 + 2t^2 + t, 2t^3 - 4t \rangle$. Find the slope of the path of the particle at $t = 3$.

$$f'(t) = \langle 3t^2 + 4t + 1, 6t^2 - 4 \rangle$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} \Big|_{t=3} = \frac{6(3)^2 - 4}{3(3)^2 + 4(3) + 1} = \frac{50}{40} = \boxed{\frac{5}{4}}$$

10. The position of a particle moving in the xy -plane is defined by the vector-valued function, $f(t) = \langle t^3 - 6t^2, 2t^3 - 9t^2 - 24t \rangle$. For what value of $t \geq 0$ is the particle at rest?

$$\frac{dx}{dt} = 3t^2 - 12t$$

$$0 = 3t(t-4)$$

$$t=0 \text{ or } \underline{t=4}$$

$$\frac{dy}{dt} = 6t^2 - 18t - 24$$

$$0 = 6(t^2 - 3t - 4)$$

$$0 = (t-4)(t+1)$$

$$\underline{t=4} \text{ or } t=-1$$

At rest when $t = 4$.

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Test Prep

11. **Calculator active.** The path of a particle moving along a path in the xy -plane is given by the vector-valued function f and f' is defined by $f'(t) = \langle t^{-1}, 2ke^{kt} \rangle$ where k is a positive constant. The line $y = 4x + 5$ is parallel to the line tangent to the path of the particle at the point where $t = 2$. What is the value of k ?

$$\frac{dy}{dx} = \frac{2ke^{kt}}{t^{-1}} \Big|_{t=2} = \frac{2ke^{2k}}{2^{-1}} \rightarrow 4ke^{2k} = 4$$

$$ke^{2k} = 1$$

$$\boxed{k \approx 0.426}$$

12. At time t , $0 \leq t \leq 2\pi$, the position of a particle moving along a path in the xy -plane is given by the vector-valued function, $f(t) = \langle t \sin t, \cos 2t \rangle$. Find the slope of the path of the particle at time $t = \frac{\pi}{2}$.

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-2\sin(2t)}{\sin t + t \cos t} \Big|_{t=\frac{\pi}{2}} = \frac{-2\sin(\pi)}{\sin \frac{\pi}{2} + \frac{\pi}{2} \cos(\frac{\pi}{2})} = \boxed{0}$$