

Write your questions
and thoughts here!

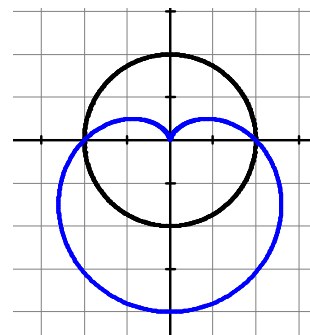
Recall area bounded by a polar curve: $A =$

Things to watch for when using more than one polar curve for area.

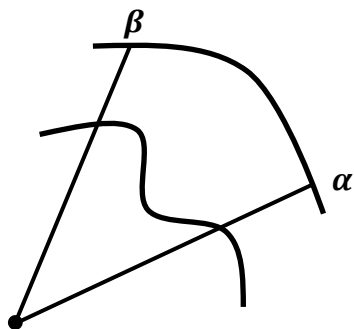
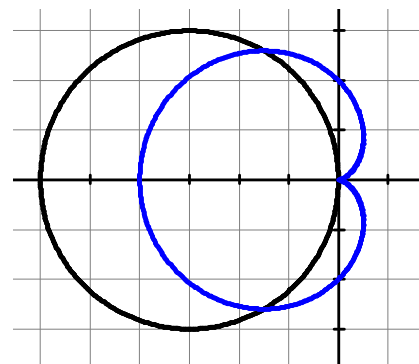
- Points of intersection
- Symmetry

1. Find the area of the region common to the polar curve $r = 4 \cos \theta$ and $r = 2 \sin \theta$.

2. Find the area of the common region to the polar graphs of $r = 2$ and $r = 2 - 2 \sin \theta$.



3. Find the area of the region common to the two polar curves $r = -6 \cos \theta$ and $r = 2 - 2 \cos \theta$.



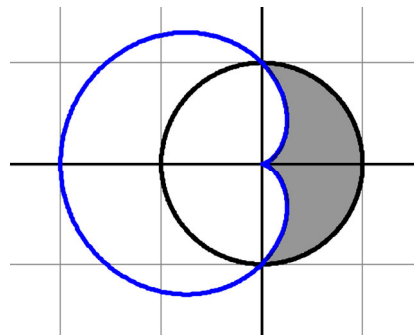
Area Bounded by Two Polar Curves

$$A =$$

$$A =$$

Write your questions and thoughts here!

4. Find the area of the region bounded by the two polar curves $r = 1$ and $r = 1 - \cos \theta$ as shown in the graph below.



9.9 Area Bounded by Two Polar Curves

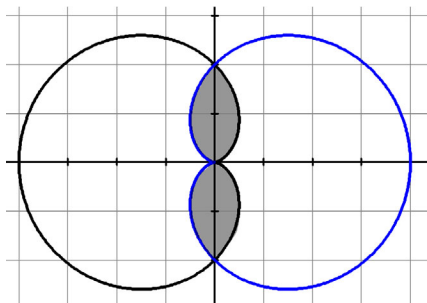
Calculus

1. Find the area of the common interior of the polar graphs $r = 4 \sin 2\theta$ and $r = 2$.

2. Find the area of the common interior of the polar graphs $r = 2 \cos \theta$ and $r = 2 \sin \theta$.

Practice

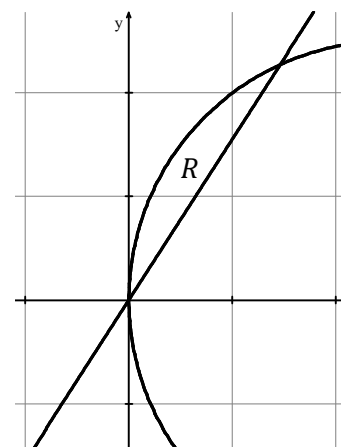
3. The polar curves $r = 2 - 2 \cos \theta$ and $r = 2 + 2 \cos \theta$ are shown below.



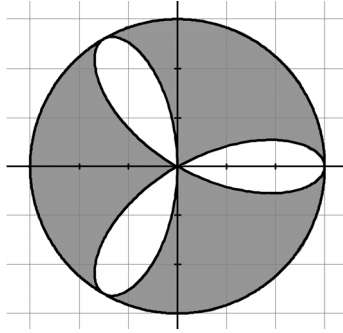
Which of the following gives the total area of the shaded regions?

- A. $\int_0^\pi (2 + 2 \cos \theta)^2 d\theta$
 B. $\int_{\pi/2}^\pi (2 + 2 \cos \theta)^2 d\theta$
 C. $8 \int_0^{\pi/2} (1 - \cos \theta)^2 d\theta$
 D. $\int_0^{\pi/2} ((2 - 2 \cos \theta)^2 + (2 + 2 \cos \theta)^2) d\theta$

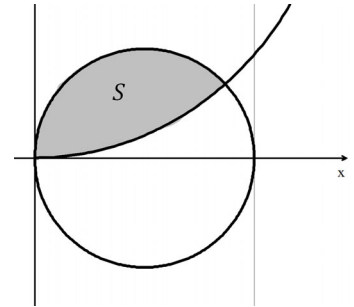
4. Let R be the region in the first quadrant that is bounded above by the polar curve $r = 5 \cos \theta$ and below by the line $\theta = 1$, as shown in the figure below. What is the area of R ?



5. The figure below shows the graphs of the polar curves $r = 3 \cos 3\theta$ and $r = 3$. What is the sum of the areas of the shaded regions?



6. Let S be the region in the 1st Quadrant bounded above by the graph of the polar curve $r = \cos \theta$ and bounded below by the graph of the polar curve $r = \frac{5}{2}\theta$, as shown in the figure above. The two curves intersect when $\theta = 0.373$. What is the area of S ?



7. Find the area inside the polar curve $r = 2 \cos \theta$ and outside the polar curve $r = 1$.

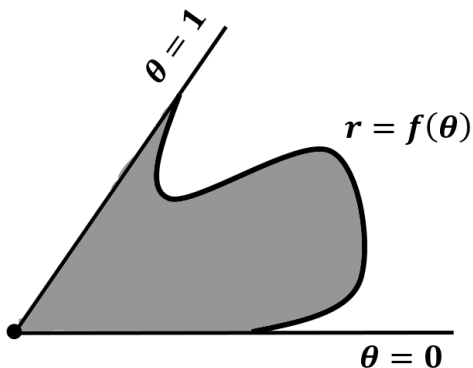
8. Write an integral expression that represents the area of the region outside the polar curve $r = 3 + 2 \sin \theta$ and inside the polar curve $r = 2$.

9. What is the total area outside the polar curve $r = 5 \cos 2\theta$ and inside the polar curve $r = 5$?

10. Find the area of the common interior of the polar curves $r = 4 \sin \theta$ and $r = 2$.

9.9 Area Bounded by Two Polar Curves

11.



θ	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
r	1	3	5	4	2

No calculator! Let R be the region bounded by the graph of the polar curve $r = f(\theta)$ and the lines $\theta = 0$ and $\theta = 1$, as shaded in the figure above. The table above gives values of the polar function $r = f(\theta)$ at selected values of θ . What is the approximation for the area of region R using a right Riemann sum with the four subintervals indicated by the data in the table?