Name: _____ Date:_____ **Corrective Assignment**

# End of Unit 5 CA – Analytical Applications of Differentiation

1. **Calculator active problem.** The first derivative of the function $f$ is given by
$$f'(x) = -2 + x + 3e^{-\cos(4x)}$$
How many points of inflection does the graph of $f$ have on the interval $0 < x < \pi$?

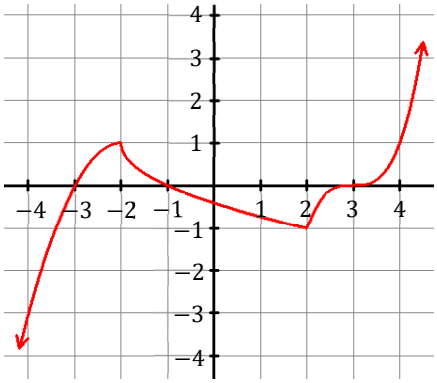
2. **Calculator active problem.** The rate of money in a particular mutual fund is represented by $m(t) = \sin\left(\frac{e}{3}\right)^t$ thousand dollars per year where $t$ is measured in years. Is the amount of money from this mutual fund increasing or decreasing at time $t = 4$ years? Justify your answer.

3. A particle is traveling along the $y$-axis and its position from the origin can be modeled by
$$y(t) = 6t - 2t^3 + 10$$
where $y$ is meters and $t$ is minutes.
   a. On the interval $0 \le t \le 2$, when is the particle farthest above the origin.

   b. On the interval $0 \le t \le 2$, what is the particle's maximum speed?

4. A rectangle is formed with the base on the $x$-axis and the top corners on the function $y = 36 - x^2$. What length and width should the rectangle have so that its area is a maximum?

5. The graph shows the derivative of $f$, $f'$. Identify the intervals when $f$ is increasing and decreasing. Include a justification statement.




Increasing: Decreasing:

6. For the table below, selected values of $x$ and $f(x)$ are given. Assume that $f'(x)$ and $f''(x)$ do not change signs.

| $x$ | $f(x)$ |
|---|---|
| 0 | $-10$ |
| 1 | $-8$ |
| 2 | $-5$ |
| 3 | $-1$ |

a. Is $f(x)$ increasing or decreasing?

b. Is $f(x)$ concave up or concave down?

7. Given the function $g(x) = -x^4 + 2x^2 - 1$, find the interval(s) when $g$ is **concave up** and **decreasing** at the same time.

8. The Mean Value Theorem can be applied to which of the following function on the closed interval $[0, 5]$?

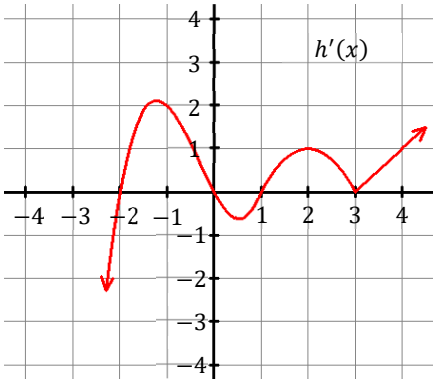
(A) $f(x) = \frac{x-3}{x+3}$

(B) $f(x) = (x-1)^{\frac{2}{3}}$

(C) $f(x) = \frac{x+3}{x-3}$

(D) $f(x) = |x-4|$

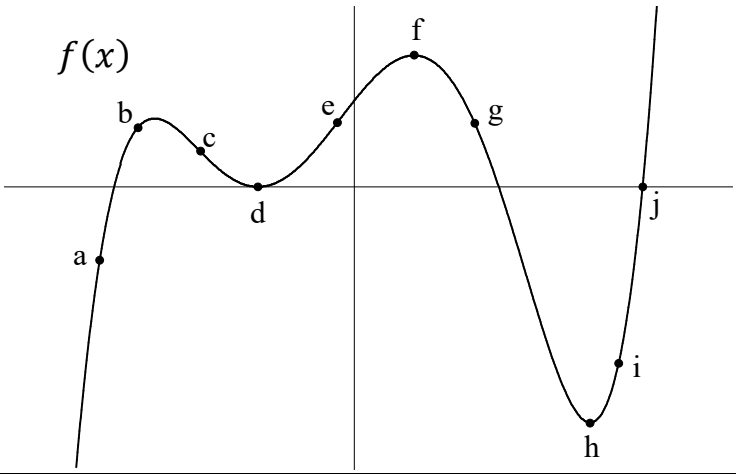
9. To the right is the graph of $h'(x)$. Identify all extrema of $h(x)$. No justification necessary on this problem.




10. The derivative of $g$ is given by $g'(x) = (5 - x)x^{-3}$ for $x > 0$. Find all relative extrema and justify your conclusions.

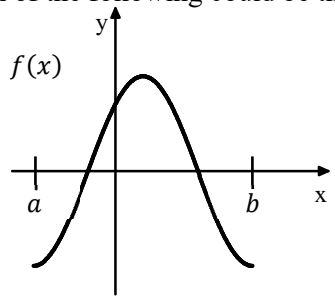
11. Consider the function $f$ defined by $f(x) = e^x \sin x$ with domain $[0, 2\pi]$. Find the absolute maximum and minimum values of $f(x)$.

12. Using the figure below, complete the chart by indicating whether each value is positive (+), negative (-), or zero (0) at the indicated points. For these problems, if the point appears to be a max or min, assume it is. If it appears to be a point of inflection, assume it is.

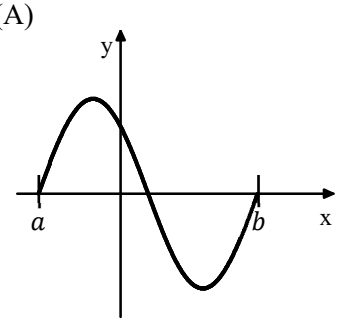



| $x$ | $a$ | $b$ | $c$ | $d$ | $e$ | $f$ | $g$ | $h$ | $i$ | $j$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | | | | | | | | | | |
| $f'(x)$ | | | | | | | | | | |
| $f''(x)$ | | | | | | | | | | |

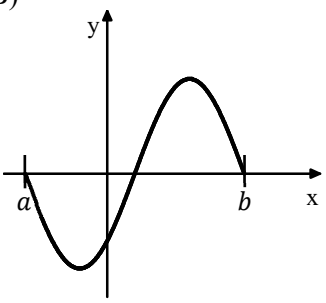
13. The graph of $f$ is shown below. Which of the following could be the graph of the derivative of $f$?



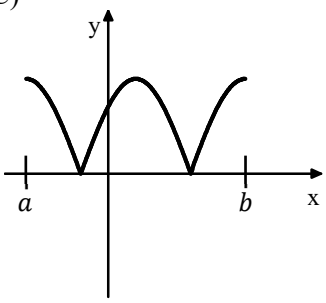

(A)



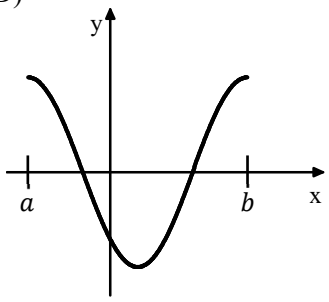

(B)



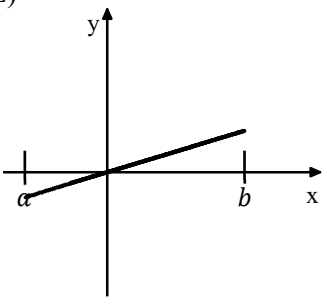

(C)




(D)




(E)




## Answers

| | | | | |
|---|---|---|---|---|
| 1. 4 | 2. Increasing because the rate $m(4)$ is positive. $m(4) \approx 0.3838$ | 3a. $y(0) = 10$ $\mathbf{y(1) = 14}$ $y(2) = 6$ At $t = 1$ minutes | 3b. $y'(0) = 6$ $y'(2) = -18$ 18 meters / minute | 4. $2\sqrt{12} \times 24$ |

| | | | | |
|---|---|---|---|---|
| 5. Increasing on the interval $(-3, -1)$ and $(3, \infty)$. Decreasing on the interval $(-\infty, -3)$ and $(-1, 3)$. | 6a. Increasing | 6b. Concave up | 7. $\left(-\sqrt{\frac{1}{3}}, 0\right)$ | 8. A |

| | | |
|---|---|---|
| 9. Min at $x = -2$ and $x = 1$. Max at $x = 0$. | 10. Relative maximum at $x = 5$ because $g'$ changes sign from positive to negative. | 11. $g(0) = 0$ $\boldsymbol{g\left(\frac{3\pi}{4}\right) = e^{\frac{3\pi}{4}}\left(\frac{\sqrt{2}}{2}\right)}$ **ABS MAX** $\boldsymbol{g\left(\frac{7\pi}{4}\right) = -e^{\frac{7\pi}{4}}\left(\frac{\sqrt{2}}{2}\right)}$ **ABS MIN** $g(2\pi) = 0$ |

12.

| $x$ | $a$ | $b$ | $c$ | $d$ | $e$ | $f$ | $g$ | $h$ | $i$ | $j$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | − | + | + | 0 | + | + | + | − | − | 0 |
| $f'(x)$ | + | + | − | 0 | + | 0 | − | 0 | + | + |
| $f''(x)$ | − | − | 0 | + | 0 | − | − | + | + | + |

13. A