Name: ______________________________ Date:______________ Period: _____ **Review**

Mid-Unit 5 Review – Analytical Applications of Differentiation

Lessons 5.1 through 5.7

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you must study all packets from Unit 5.

1. If $y = -2x^2 + 4x + 3$ apply the Mean Value Theorem to find when the instantaneous rate of change will equal the average rate of change on the interval $[1, 3]$.

Avg $\frac{y(3) - y(1)}{3 - 1} = \frac{-3 - 5}{2} = -4$

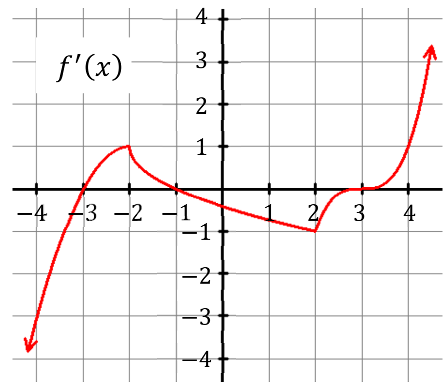
$y' = -4x + 4$

$-4x + 4 = -4$

$-4x = -8$

$\boxed{x = 2}$

2. Below is the graph of f'. Find all relative extrema of f and justify.



Relative minimum at $x = -3$ and $x = 3$ because f' changes sign from negative to positive.

Rel. max at $x = -1$ b/c f' changes sign from pos to neg.

3. The derivative of g is given by $g'(x) = 6x^2 - 6$. Use the First Derivative Test to find all relative extrema and justify your conclusions.

$6x^2 - 6 = 0$

$x^2 = 1$

$x = \pm 1$

x	$(-\infty, -1)$	-1	$(-1, 1)$	1	$(1, \infty)$
$f'(x)$	$+$	0	$-$	0	$+$

Rel. max at $x = -1$ b/c f' changes from pos. to neg.

Rel. min at $x = 1$ b/c f' changes sign from neg. to pos.

4. What is the minimum value of $f(x) = xe^{\frac{x}{3}}$?

$f'(x) = e^{x/3} + \frac{1}{3}xe^{x/3}$

$e^{x/3}\left(1 + \frac{1}{3}x\right) = 0$

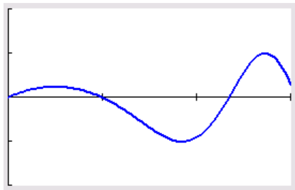
$x = -3$

x	$(-\infty, -3)$	-3	$(3, \infty)$
f'	$-$	0	$+$

$f(-3) = -3e^{-1}$

$\boxed{-\frac{3}{e}}$

5. **Calculator active problem.** The derivative of f is defined by $f'(x) = \sin(x - x^2)$ for $0 \le x \le 3$. On what interval(s) is f decreasing?



$\boxed{1 < x < 2.3416}$

6. What is the absolute maximum value AND the absolute minimum value of the function $g(x) = x^3 - 12x$ on the closed interval $[0, 4]$.

$g'(x) = 3x^2 - 12$

$3x^2 - 12 = 0$

$x^2 = 4$

$x = \pm 2$

$g(-2)$ is outside the interval

$g(0) = 0$

$g(2) = -16$ ← Abs min

$g(4) = 16$ ← Abs max

7. Use the 2[nd] Derivative Test to find x-values of the extrema of $g(x) = 2\cos x - x$ on the interval $(0, 2\pi)$ and justify your answer.

$g'(x) = -2\sin x - 1 = 0$

$\sin x = -\frac{1}{2}$

$x = \frac{7\pi}{6}$ $\quad x = \frac{11\pi}{6}$

$g''(x) = -2\cos x$

$g''\left(\frac{7\pi}{6}\right) = -2\left(-\frac{\sqrt{3}}{2}\right) > 0$

$g''\left(\frac{11\pi}{6}\right) = -2\left(\frac{\sqrt{3}}{2}\right) < 0$

Rel min at $x = \frac{7\pi}{6}$ because $g'\left(\frac{7\pi}{6}\right) = 0$ and $g''\left(\frac{7\pi}{6}\right) > 0$.

Rel. max at $x = \frac{11\pi}{6}$ because $g'\left(\frac{11\pi}{6}\right) = 0$ and $g''\left(\frac{7\pi}{6}\right) < 0$.

8. Find the intervals of concavity for the function $f(x) = x^4 + 4x^3 - 18x^2 - 4x + 7$

$f'(x) = 4x^3 + 12x^2 - 36x - 4$

$f''(x) = 12x^2 + 24x - 36$

$12(x^2 + 2x - 3) = 0$

$12(x + 3)(x - 1) = 0$

$x = -3 \quad x = 1$ ← Possible pts of inflection

x	$(-\infty, -3)$	-3	$(-3, 1)$	1	$(1, \infty)$
$f''(x)$	+	0	−	0	+

f is concave up on $(-\infty, -3)$ and $(1, \infty)$ because $f''(x) > 0$.

f is concave down on $(-3, 1)$ because $f''(x) < 0$.