Name: Solutions Date:______________ Period: _____

Review

Mid-Unit 8 Review – Applications of Integration

Lessons 8.1 through 8.6

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you must study all packets from Unit 8.

Average Rate of Change	Mean Value Theorem	Average Value of a Function
$\dfrac{f(b)-f(a)}{b-a}$	$f'(c) = \dfrac{f(b)-f(a)}{b-a}$	$\dfrac{1}{b-a}\int_a^b f(x)\,dx$

Find the average value of each function on the given interval.

1. $f(x) = x^3$ on $[0,2]$

$\frac{1}{2-0}\int_0^2 x^3\,dx$

$\frac{1}{2}\cdot\frac{x^4}{4}\Big|_0^2$

$\frac{1}{8}\left[2^4 - 0\right]$

$\frac{16}{8} = \boxed{2}$

2. $f(x) = \frac{1}{x}$ on $[1, e]$

$\frac{1}{e-1}\int_1^e \frac{1}{x}\,dx$

$\frac{1}{e-1}\left[\ln x\right]_1^e$

$\frac{1}{e-1}\left[\ln e - \ln 1\right]$

$\boxed{\frac{1}{e-1}}$

$$\int \text{rate of change} = \text{net change}$$

$$\int \text{velocity} = \text{displacement} \qquad \int |\text{velocity}| = \text{total distance}$$

3. A particle's velocity is given by $v(t) = 6t^2 - 18t + 12$, where t is measured in seconds, v is measured in feet per second, and $s(t)$ represents the particle's position.

(a) If $s(1) = 3$, what is the value of $s(2)$?

$3 + \int_1^2 (6t^2 - 18t + 12)\,dt$

$3 + \left[2t^3 - 9t^2 + 12t\right]\Big|_1^2$

$3 + (16 - 36 + 24) - (2 - 9 + 12) = 3 + 4 - 5 = \boxed{2}$

(b) What is the net change in distance over the first 3 seconds?

$\int_0^3 (6t^2 - 18t + 12)\,dt$

$\left[2t^3 - 9t^2 + 12t\right]\Big|_0^3$

$\left[2(27) - 9(9) + 36\right] - [0] = 54 - 81 + 36 = \boxed{9 \text{ feet}}$

(c) What is the total distance traveled by the particle during the first 2 seconds? Show the set up AND your answer.

$v(t) = 0$

$6(t^2 - 3t + 2) = 0$

$(t-2)(t-1) = 0$

$t = 2 \quad t = 1$

$\left|\int_0^1 (6t^2 - 18t + 12)\,dt\right| + \left|\int_1^2 (6t^2 - 18t + 12)\,dt\right|$

$|5| + |-1| = \boxed{6}$

4. A particle moves along a coordinate line. Its acceleration function is $a(t) = 6t - 22$ for $t \geq 0$. If $v(0) = 24$ find the velocity at $t = 4$.

$v(t) = \int 6t - 22 \, dt$

$v(t) = 3t^2 - 22t + C$

$24 = 0 - 0 + C$

$24 = C$

$v(4) = 3(16) - 22(4) + 24$

$v(4) = 48 - 88 + 24$

$\boxed{v(4) = -16}$

5. A particle's velocity is given by $v(t) = \cos t$, where t is measured in months, v is measured in kilometers per month, and $s(t)$ represents the particle's position.

(a) If $s\left(\frac{\pi}{6}\right) = 10$, what is the value of $s\left(\frac{3\pi}{2}\right)$?

$s\left(\frac{3\pi}{2}\right) = 10 + \int_{\pi/6}^{3\pi/2} \cos t \, dt$

$= 10 + \sin t \Big|_{\pi/6}^{3\pi/2}$

$10 + \sin\left(\frac{3\pi}{2}\right) - \sin\left(\frac{\pi}{6}\right)$

$10 + (-1) - \left(\frac{1}{2}\right)$

$\boxed{8.5 \text{ km}}$

(b) What is the net change in distance over the first π months?

$\int_0^{\pi} \cos t \, dt$

$\sin t \Big|_0^{\pi} = \sin \pi - \sin 0 = 0 - 0 = \boxed{0}$

(c) What is the total distance traveled by the particle during the first π months? Show the set up AND your answer.

$v(t) = 0$

$\cos t = 0$

$t = \frac{\pi}{2}$

$\int_0^{\pi/2} \cos t \, dt + \int_{\pi/2}^{\pi} \cos t \, dt$

$\sin t \Big|_0^{\pi/2} + \sin t \Big|_{\pi/2}^{\pi}$

$1 - 0 + 0 - (-1) = 1 + 1 = \boxed{2}$

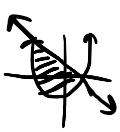
6. Find the area between the two curves $y = x^2 - 4$ and $y = 2 - x$.

$x^2 - 4 = 2 - x$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$x = -3 \quad x = 2$



$\int_{-3}^{2} [(2-x) - (x^2-4)] \, dx$

$\int_{-3}^{2} (-x^2 - x + 6) \, dx$

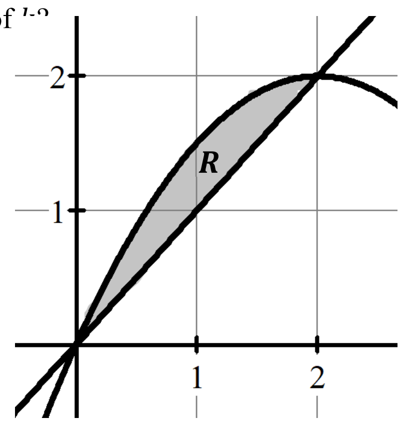
$-\frac{x^3}{3} - \frac{x^2}{2} + 6x \Big|_{-3}^{2}$

$\left(-\frac{8}{3} - \frac{4}{2} + 12\right) - \left(9 - \frac{9}{2} - 18\right)$

$-\frac{8}{3} + 10 + 9 + \frac{9}{2}$

$\approx \boxed{20.833}$

7. **Calculator active.** Let R be the region bounded by the graphs $y = 2x - \frac{1}{2}x^2$ and $y = x$ as sown in the figure. If the line $x = k$ divides R into two regions of equal area, what is the value of k?



$\int_0^k \left(2x - \frac{1}{2}x^2 - x\right) dx = \int_k^2 \left(2x - \frac{1}{2}x^2 - x\right) dx$

$\int_0^k \left(x - \frac{1}{2}x^2\right) dx = \int_k^2 \left(x - \frac{1}{2}x^2\right) dx$

$\left[\frac{x^2}{2} - \frac{x^3}{6}\right] \Big|_0^k = \left[\frac{x^2}{2} - \frac{x^3}{6}\right] \Big|_k^2$

$\frac{k^2}{2} - \frac{k^3}{6} - 0 = 2 - \frac{4}{3} - \left(\frac{k^2}{2} - \frac{k^3}{6}\right)$

$k^2 - \frac{k^3}{3} = 2 - \frac{4}{3}$

$\boxed{k = 1}$

8. **Calculator active.** A 10,000-liter tank of water is filled to capacity. At time $t = 0$, water begins to drain out of the tank at a rate modeled by $r(t)$, measured in liters per hour, where r is given by the piecewise-defined function

$$r(t) = \frac{400t}{t+2} \text{ for } 0 \le t \le 6.$$

a. Find $\int_0^6 r(t)\,dt$

$\boxed{1290.9645}$

b. Explain the meaning of your answer to part *a* in the context of this problem.

1,290.964 liters have drained out of the tank during the first six hours.

c. Write, but do not solve, an equation involving an integral to find the time A when the amount of water in the tank is 8,000 liters.

$$\boxed{10{,}000 - \int_0^A \frac{400t}{t+2}\,dt = 8{,}000}$$

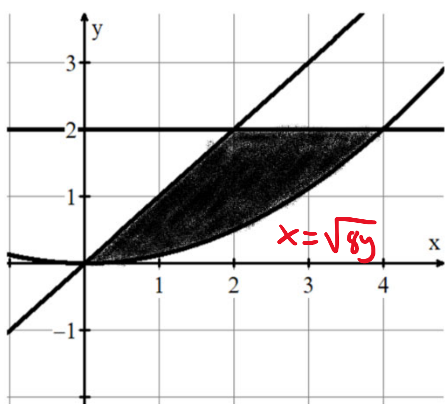
It is "minus" because $r(t)$ represents the rate of water draining from the tank.

Set up the integral(s) that give the area of the region bounded by the given equations. Show the equivalent set up with respect to x as well as with respect to y.

9. $y = x,\ \ y = \frac{x^2}{8},\ \ y = 2$

with respect to x

$$\boxed{\int_0^2 \left(x - \frac{x^2}{8}\right)dx + \int_2^4 \left(2 - \frac{x^2}{8}\right)dx}$$



with respect to y

$x = y$ $\quad 8y = x^2$

$\pm\sqrt{8y} = x$

$$\boxed{\int_0^2 \left(\sqrt{8y} - y\right)dy}$$