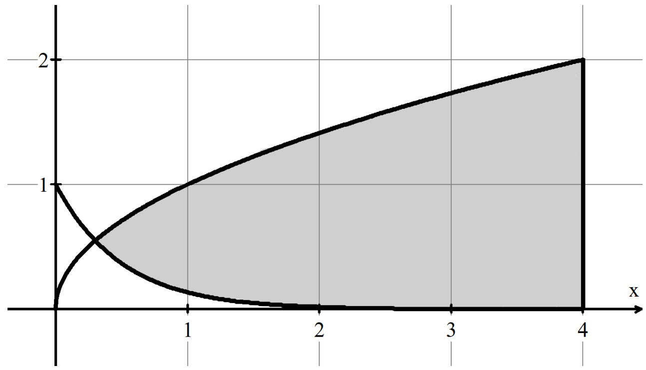
Name: Solutions Date:______________ Period: _____ **Review**

End-of-Unit 8 Review – Applications of Integration

Lessons 8.7 through 8.12

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you must study all packets from Unit 8.



Calculator active. Let R be the shaded region bounded by the graphs of $y = \sqrt{x}$ and $y = e^{-2x}$ and the vertical line $x = 4$ as shown in the figure above.

$\sqrt{x} = e^{-2x}$ when $x = 0.300542 \to a$

1. Find the area of R

$$A = \int_a^4 \left(\sqrt{x} - e^{-2x}\right) dx$$

$$A = 4.9495$$

2. Find the volume of the solid generated when R is revolved about the horizontal line $y = 2$.

$$V = \pi \int_a^4 \left[\left(2 - e^{-2x}\right)^2 - \left(2 - \sqrt{x}\right)^2\right] dx$$

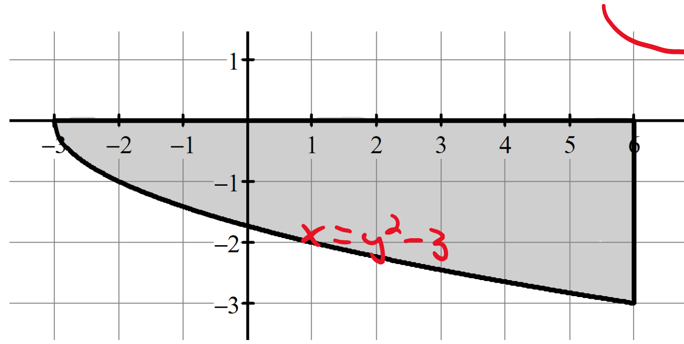
$$V \simeq 37.443$$

3. The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a rectangle whose height is 4 times the length of its base in region R. Find the volume of this solid.

$$V = \int_a^4 4\left(\sqrt{x} - e^{-2x}\right)^2 dx$$

$$V \simeq 30.2365$$

Calculator active. Let T be the region enclosed by the graph of $y = -\sqrt{x+3}$, the vertical line $x = 6$, and the x-axis.



$-y = \sqrt{x+3}$

$y^2 = x+3$

$x = y^2 - 3$

4. Find the area of T.

$$A = \int_{-3}^{6} \left(0 - -\sqrt{x+3}\right) dx$$

$$A = 18$$

5. The region T is the base of a solid. For this solid, each cross section perpendicular to the y-axis is a semicircle. Find the volume of this solid.

$$V = \int_{-3}^{0} \frac{\pi}{2}\left(\frac{6-(y^2-3)}{2}\right)^2 dy$$

$$V \approx 50.8938$$

6. Find the volume of the solid generated when T is revolved about the horizontal line $y = -3$.

$$V = \pi\int_{-3}^{6}\left[(-3)^2 - \left(-3 - -\sqrt{x+3}\right)^2\right] dx$$

$$V \approx 212.0575$$

7. Find the volume of the solid generated when T is revolved about the vertical line $x = 6$.

$$V = \pi\int_{-3}^{0}\left[(y^2-3-6)^2\right] dy$$

Shift left 6

$$V \approx 407.150$$

8. Find an integral that is equal to the length of the curve $f(x) = 3x^4 + x^2 - 2x + 1$ from the points $(0,1)$ to $(2, 49)$. **Do Not Evaluate.**

$$f'(x) = 12x^3 + 2x - 2$$

$$\int_0^2 \sqrt{1 + (12x^3 + 2x - 2)^2}\, dx$$

9. The length of the curve $y = 2x^3$ from $x = 1$ to $x = 5$ is given by

$$y' = 6x^2$$

$$(y')^2 = (6x^2)^2 = 36x^4$$

A. $\int_1^5 \sqrt{1 + 4x^4}\, dx$

B. $\int_1^5 \sqrt{1 + 6x^2}\, dx$

C. $\int_1^5 \sqrt{1 + 36x^4}\, dx$

D. $\int_1^5 \sqrt{1 + x^6}\, dx$

E. $\int_1^5 \sqrt{1 + 36x^3}\, dx$