Name: Solutions Date:______________ Period: _____ **Review**

End-of-Unit 6 Review – Integration and Accumulation of Change

Lessons 6.6 through 6.14

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you must study all packets from Unit 6.

Find the value of the definite integral.

1. $\int_{-2}^{-1}\left(\frac{1}{x^2} + x^2 - 5x\right)dx$

$-\frac{1}{x} + \frac{x^3}{3} - \frac{5x^2}{2} \Big|_{-2}^{-1}$

$\left[-\frac{1}{(-1)} + \frac{-1}{3} - \frac{5}{2}\right] - \left[-\frac{1}{(-2)} - \frac{8}{3} - \frac{20}{2}\right]$

$\left[1 - \frac{1}{3} - \frac{5}{2}\right] - \left[\frac{1}{2} - \frac{8}{3} - 10\right]$

$11 + \frac{7}{3} - \frac{6}{2}$

$8 + \frac{7}{3}$

$\frac{24}{3} + \frac{7}{3} = \boxed{\frac{31}{3}}$

2. $\int_{-1}^{8}\left(x^{2/3} - x\right)dx$

$\frac{x^{5/3}}{5/3} - \frac{x^2}{2} \Big|_{-1}^{8}$

$\frac{3}{5}\left(\sqrt[3]{x}\right)^5 - \frac{1}{2}x^2 \Big|_{-1}^{8}$

$\left[\frac{3}{5}(2)^5 - 32\right] - \left[-\frac{3}{5} - \frac{1}{2}\right]$

$\left[\frac{96}{5} - 32\right] - \left[-\frac{6}{10} - \frac{5}{10}\right]$

$\frac{192}{10} - \frac{320}{10} + \frac{11}{10}$

$\boxed{-\frac{117}{10}}$

3. $\int_0^{\pi}(x - \sin x)\,dx$

$\frac{x^2}{2} + \cos x \Big|_0^{\pi}$

$\left[\frac{\pi^2}{2} + (-1)\right] - \left[0 + 1\right]$

$\frac{\pi^2}{2} - 1 - 1$

$\boxed{\frac{\pi^2}{2} - 2}$

4. $\int_{-1}^{1} x\sqrt{1 - x^2}\,dx$

$u = 1 - x^2$

$\frac{du}{-2x} = dx$

$\int_0^0 x\sqrt{u}\,\frac{du}{-2x}$

$-\frac{1}{2}\int_0^0 \sqrt{u}\,du$

$\boxed{0}$

lower bound = upper bound

5. $\int_0^{\frac{\pi}{6}} \frac{\sin(2x)}{\cos^2(2x)}\,dx$

$u = \cos(2x)$

$du = -\sin(2x)\cdot 2dx$

$\frac{du}{-2\sin(2x)} = dx$

$\int_1^{1/2} \frac{\sin(2x)}{u^2}\left(\frac{du}{-2\sin(2x)}\right)$

$-\frac{1}{2}\int_1^{1/2} u^{-2}\,du$

$-\frac{1}{2}\left[\frac{u^{-1}}{-1}\right] \Big|_1^{1/2}$

$\frac{1}{2}\left[\frac{1}{1/2} - \frac{1}{1}\right] = \frac{1}{2}\left[1\right] = \boxed{\frac{1}{2}}$

6. $\int_e^{e^2} \frac{1}{x\ln x}\,dx$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$x\,du = dx$

$\int_1^2 \frac{1}{xu}\cdot(x\,du)$

$\int_1^2 \frac{1}{u}\,du$

$\ln|u| \Big|_1^2$

$\ln 2 - \ln 1$

$\boxed{\ln 2}$

7. If $\int_{-5}^{2} f(x)dx = -17$ and $\int_{5}^{2} f(x)dx = 4$, what is the value of $\int_{-5}^{5} f(x)dx$?

$\int_{-5}^{5} f(x)\,dx = \int_{-5}^{2} f(x)\,dx + \int_{2}^{5} f(x)\,dx$

$= (-17) + (-4)$

(A) -21 (B) -13 (C) 0 (D) 13 (E) 21

Find the following indefinite integrals.

8. $\int \left(\frac{x^2-x+5}{x}\right) dx$

$\int \left(x - 1 + \frac{5}{x}\right) dx$

$\boxed{\frac{x^2}{2} - x + 5\ln x + C}$

9. $\int \sec x \tan x \, dx$

$\boxed{\sec x + C}$

10. $\int (e^x + 2^x)\, dx$

$\boxed{e^x + \frac{1}{\ln 2} 2^x + C}$

11. $\int \left(\frac{1}{x} + \frac{1}{x^3}\right) dx$

$\ln|x| + \frac{x^{-2}}{-2} + C$

$\boxed{\ln|x| - \frac{1}{2x^2} + C}$

12. $\int \sqrt{x}\left(x - \frac{4}{x}\right) dx$

$\int x^{3/2} - 4x^{-1/2} \, dx$

$\frac{x^{5/2}}{5/2} - \frac{4x^{1/2}}{1/2} + C$

$\boxed{\frac{2}{5}x^{5/2} - 8\sqrt{x} + C}$

13. $\int \frac{50x^3-55x^2-26x+33}{10x-7} dx$

$$\begin{array}{r} 5x^2 - 2x - 4 + \frac{5}{10x-7} \\ 10x-7 \enclose{longdiv}{50x^3 - 55x^2 - 26x + 33} \\ -(50x^3 - 35x^2) \\ -20x^2 - 26x + 33 \\ -(-20x^2 + 14x) \\ -40x + 33 \\ -(-40x + 28) \\ 5 \end{array}$$

$\int 5x^2 - 2x - 4 + \frac{5}{\underbrace{10x-7}} \, dx$

$u = 10x - 7$

$\frac{du}{10} = dx$

$\frac{5x^3}{3} - \frac{2x^2}{2} - 4x + \frac{5}{10}\ln|10x-7| + C$

$\boxed{\frac{5}{3}x^3 - x^2 - 4x + \frac{1}{2}\ln|10x-7| + C}$

14. $\int \frac{1}{x^2+2x+2} dx$

$(x^2 + 2x + 1) + 2 - 1$

$\int \frac{1}{(x+1)^2 + 1} dx$

$\boxed{\tan^{-1}(x+1) + C}$

15. **Calculator active problem.** If $f'(x) = \sin(e^x)$ and $f(0) = 5.7$, then $f(2) =$

$\boxed{5.7 + \int_0^2 \sin(e^x)\, dx \approx 6.2509}$