Calculus

Write your questions and thoughts here!

1.1 Limits Graphically

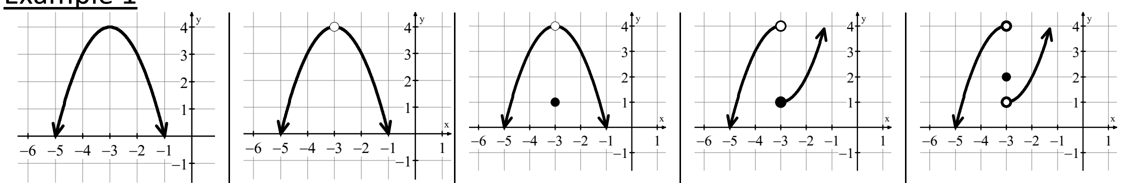
Name:____________________________

Notes

What is a limit?

A ***limit*** is the ___________ a function _______________ from *both* the left and the right side of a given ____________.

Example 1



$f(-3) =$	$f(-3) =$	$f(-3) =$	$f(-3) =$	$f(-3) =$
$\lim_{x\to -3} f(x) =$	$\lim_{x\to -3} f(x) =$	$\lim_{x\to -3} f(x) =$	$\lim_{x\to -3} f(x) =$	$\lim_{x\to -3} f(x) =$

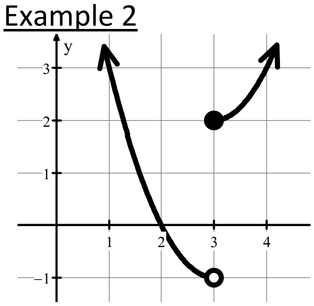
Limit: (*geeky math definition for Mr. Kelly*)

Given a function f, the limit of $f(x)$ as x approaches c is a real number R if $f(x)$ can be made arbitrarily close to R by taking x sufficiently close to c (but not equal to c). If the limit exists and is a real number, then the common notation is $\boldsymbol{\lim_{x\to c} f(x) = R}$.

What is a *one-sided* limit?

A ***one-sided limit*** is the ____________ a function approaches as you approach a given ____________ from either the ______ or ______ side.

Example 2



"The limit of f as x approaches 3 from the left side is -1."	$\lim_{x\to 3^-} f(x) = -1$
"The limit of f as x approaches 3 from the right side is 2."	$\lim_{x\to 3^+} f(x) = 2$

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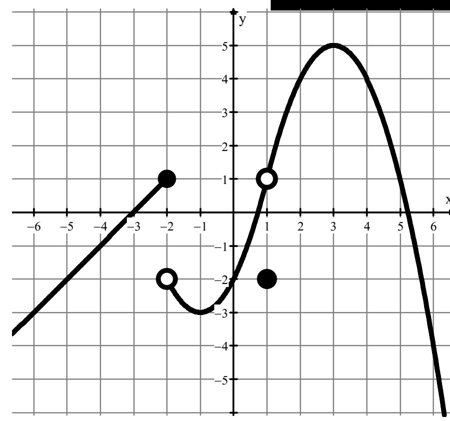
1.1 Limits Graphically

Notes

Example 3

a. $\lim\limits_{x\to -2^-} f(x) =$	b. $\lim\limits_{x\to -2^+} f(x) =$	c. $\lim\limits_{x\to -2} f(x) =$
d. $\lim\limits_{x\to 1} f(x) =$	e. $\lim\limits_{x\to 0} f(x) =$	f. $\lim\limits_{x\to 3^-} f(x) =$
g. $\lim\limits_{x\to -1} f(x) =$	h. $\lim\limits_{x\to -3} f(x) =$	i. $f(-2) =$

j. $f(1) =$



When does a limit not exist?

1.
2.
3.

Example 4

Sketch a graph of a function g that satisfies all of the following conditions.

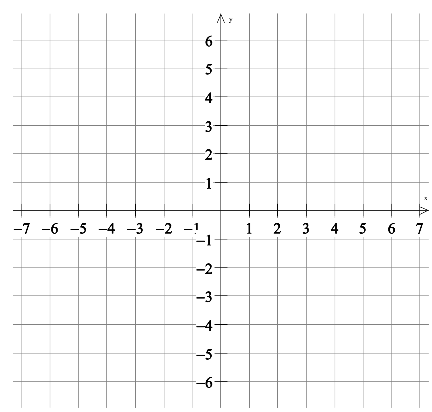
a. $g(3) = -1$

b. $\lim\limits_{x\to 3} g(x) = 4$

c. $\lim\limits_{x\to -2^+} g(x) = 1$

d. g is increasing on $-2 < x < 3$

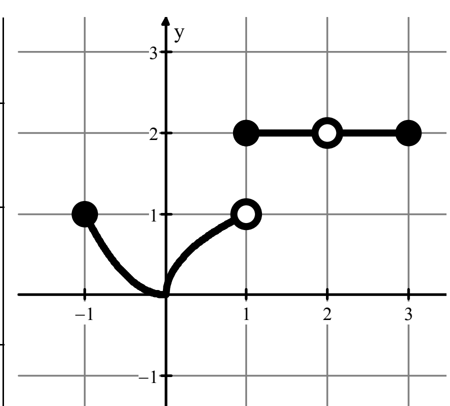
e. $\lim\limits_{x\to -2^-} g(x) > \lim\limits_{x\to -2^+} g(x)$



Example 5

Write **T** (true) or **F** (false) under each statement. Use the graph on the right.

a. $\lim\limits_{x\to -1^+} f(x) = 1$	b. $\lim\limits_{x\to 2} f(x) = 2$	c. $\lim\limits_{x\to 1^-} f(x) = 1$
d. $\lim\limits_{x\to 1^+} f(x) = 2$	e. $\lim\limits_{x\to 1} f(x) =$ does not exist	
f. $\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^+} f(x)$	g. $\lim\limits_{x\to 2} f(x) =$ does not exist	



Now summarize what you learned!

1.1 Limits Graphically

Calculus Name: ____________________________ **Practice**

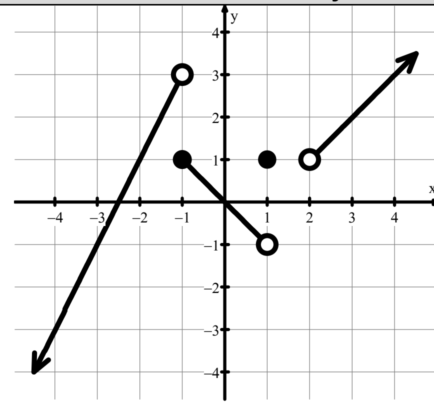
For 1-5, give the value of each statement. If the value does not exist, write "*does not exist*" or "*undefined*."

1.

a. $\lim_{x\to -1^-} f(x) =$ b. $f(1) =$ c. $\lim_{x\to 0} f(x) =$

d. $\lim_{x\to 2^+} f(x) =$ e. $f(-1) =$ f. $f(2) =$

g. $\lim_{x\to -1^+} f(x) =$ h. $\lim_{x\to 1^-} f(x) =$ i. $\lim_{x\to 2} f(x) =$

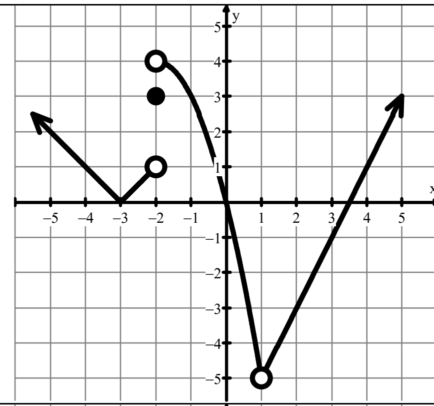


2.

a. $\lim_{x\to -3} f(x) =$ b. $f(1) =$ c. $\lim_{x\to 1} f(x) =$

d. $\lim_{x\to -2^+} f(x) =$ e. $f(3) =$ f. $\lim_{x\to -2^-} f(x) =$

g. $\lim_{x\to -2} f(x) =$ h. $f(-2) =$ i. $f(4) =$

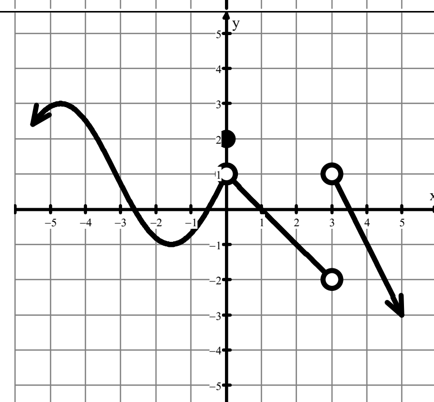


3.

a. $\lim_{x\to 3^+} f(x) =$ b. $f(3) =$ c. $\lim_{x\to 0} f(x) =$

d. $\lim_{x\to 3} f(x) =$ e. $f(0) =$ f. $\lim_{x\to 3^-} f(x) =$

g. $\lim_{x\to 0^+} f(x) =$ h. $f(1) =$ i. $f(-1.6) =$

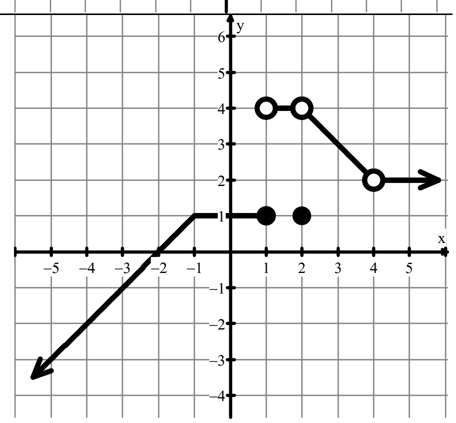


4.

a. $\lim_{x\to -1^-} f(x) =$ b. $f(2) =$ c. $\lim_{x\to 2} f(x) =$

d. $\lim_{x\to -1} f(x) =$ e. $f(4) =$ f. $\lim_{x\to 1^-} f(x) =$

g. $\lim_{x\to -1^+} f(x) =$ h. $f(1) =$ i. $\lim_{x\to 4} f(x) =$

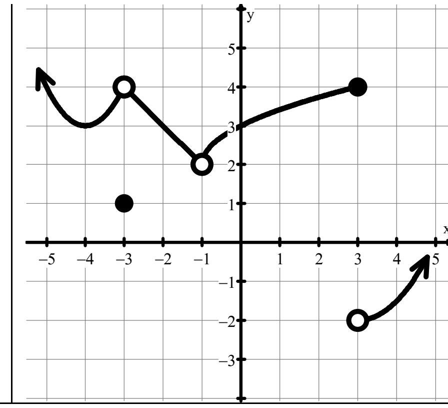


5.

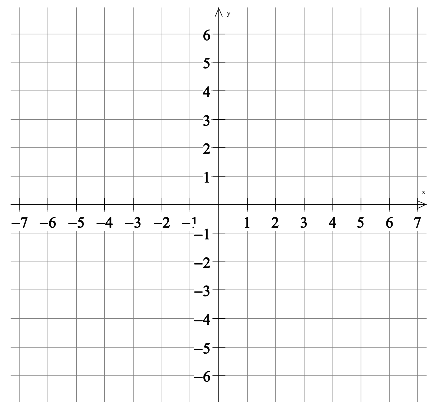
a. $\lim_{x \to 3^-} f(x) =$ b. $f(-1) =$ c. $\lim_{x \to -3} f(x) =$

d. $\lim_{x \to -1} f(x) =$ e. $f(-3) =$ f. $\lim_{x \to 3^+} f(x) =$

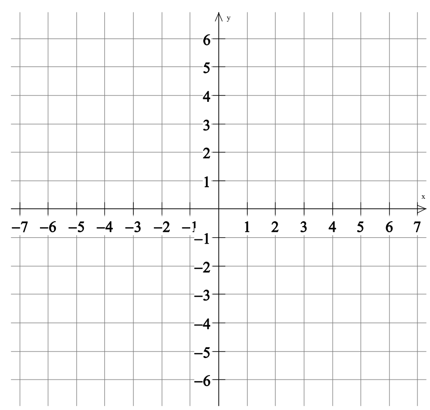
g. $f(3) =$ h. $\lim_{x \to 0} f(x) =$ i. $f(-4) =$



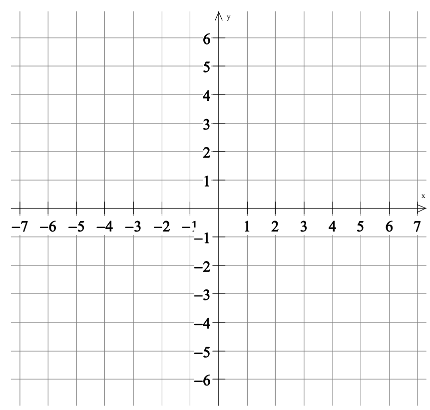
6. Sketch a graph of a function f that satisfies all of the following conditions.
   a. $f(-2) = 5$
   b. $\lim_{x \to -2} f(x) = 1$
   c. $\lim_{x \to 4^+} f(x) = 3$
   d. f is increasing on $x < -2$
   e. $\lim_{x \to 4^-} f(x) < \lim_{x \to 4^+} f(x)$



7. Sketch a graph of a function g that satisfies all of the following conditions.
   a. $g(1) = 3$
   b. $\lim_{x \to 1} g(x) = -2$
   c. $\lim_{x \to -3^+} g(x) = 5$
   d. g is increasing only on $-5 < x < -3$ and $x > 1$
   e. $\lim_{x \to -3^-} g(x) > \lim_{x \to -3^+} g(x)$



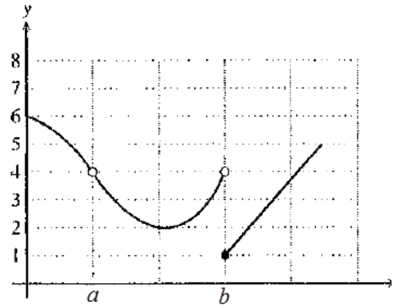
8. Sketch a graph of a function h that satisfies all of the following conditions.
   a. $\lim_{x \to 3} h(x) = h(-2) = 1$
   b. $h(3)$ is undefined.
   c. $\lim_{x \to -2^-} h(x) < \lim_{x \to -2^+} h(x)$
   d. h is constant on $-2 < x < 3$ and decreasing everywhere else.



1.1 Limits Graphically

Test Prep

1. The graph of the function f is shown. Which of the following statements about f is true?



(A) $\lim_{x\to a} f(x) = \lim_{x\to b} f(x)$

(B) $\lim_{x\to a} f(x) = 4$

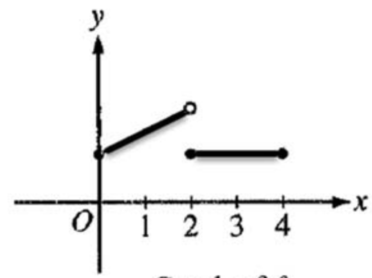
(C) $\lim_{x\to b} f(x) = 4$

(D) $\lim_{x\to b} f(x) = 1$

(E) $\lim_{x\to a} f(x)$ does not exist.

2. The figure below shows the graph of a function f with domain $0 \le x \le 4$. Which of the following statements are true?

I. $\lim_{x\to 2^-} f(x)$ exists.
II. $\lim_{x\to 2^+} f(x)$ exists.
III. $\lim_{x\to 2} f(x)$ exists.



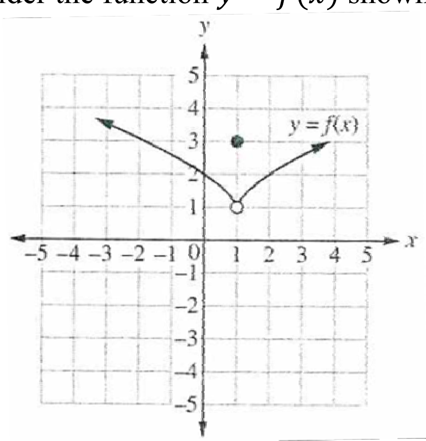
Graph of f

(A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III

3. If $[x]$ represents the greatest integer that is less than or equal to x, then $\lim_{x\to 0^-} \frac{2}{[x]} =$

(A) -2 (B) -1 (C) 0 (D) 2 (E) the limit does not exist

4. Consider the function $y = f(x)$ shown below. Which of the following statements is true?



(A) $\lim_{x\to 1} f(x) = 3$

(B) $f(1) = 1$

(C) $f(x)$ is continuous for all x.

(D) $\lim_{x\to 1} f(x) = f(1)$

(E) None of the above