Calculus

Write your questions and thoughts here!

1.2 Limits Analytically

Name:_____

Recall: What is a limit?

Notes

Finding a limit:

1.

2.

a.

b.

3.

Direct Substitution		Factor and Cancel	
1. $\lim_{x\to -1}(x^2+2x-4)$	2. $\lim_{x\to 2}\sqrt{3x-2}$	3. $\lim_{x\to 0}\frac{4x^2-5x}{x}$	4. $\lim_{x\to -7}\frac{2x^2+13x-7}{x+7}$

Rationalize	Two variables
5. $\lim_{x\to 5}\frac{\sqrt{x+4}-3}{x-5}$	6. $\lim_{h\to 0}\frac{(x+h)^2-3(x+h)-(x^2-3x)}{h}$

Write your questions and thoughts here!

1.2 Limits Analytically

Notes

Piecewise defined functions and limits

$$f(x)=\begin{cases}\sqrt{11-x}, & x<-5\\ \dfrac{x+3}{5-x^2}, & x\geq -5\end{cases}$$

$$g(x)=\begin{cases}\sqrt{10-x^2}, & x<-1\\ \dfrac{26-5x^2}{7} & -1<x\leq e\\ \ln x^3, & x>e\end{cases}$$

7. $\lim_{x\to -5^-} f(x) =$

8. $\lim_{x\to -5^+} f(x) =$

9. $\lim_{x\to -5} f(x) =$

10. $\lim_{x\to -1} g(x) =$

11. $\lim_{x\to e^+} g(x) =$

12. $\lim_{x\to e} g(x) =$

Special Trig Limits:

$$\lim_{x\to 0}\frac{\sin x}{x}= \quad \text{or} \quad \lim_{x\to 0}\frac{x}{\sin x}=$$

$$\lim_{x\to 0}\frac{1-\cos x}{x}= \quad \text{or} \quad \lim_{x\to 0}\frac{\cos x-1}{x}=$$

13. $\lim_{x\to 0}\frac{\sin 3x}{x}$

14. $\lim_{x\to 0}\frac{\tan 4x}{8x}$

15. $\lim_{x\to 0}\frac{\cos^2 x-1}{x(\cos x+1)}$

Now summarize what you learned!

1.2 Limits Analytically

Calculus

Name: _____

Practice

Evaluate each limit.

1. $\lim_{x\to 2}(x-x^2)$	2. $\lim_{x\to 5}(x+1)^2$	3. $\lim_{x\to 1}\frac{x^2-5x}{x-1}$	4. $\lim_{x\to 1}\frac{x^2+x-30}{x-1}$	5. $\lim_{x\to 0}\frac{3x}{\sin x}$
6. $\lim_{x\to 0}\frac{\sin(2x)}{3x}$	7. $\lim_{x\to 0}\frac{\sqrt{x+7}-\sqrt{7}}{x}$	8. $\lim_{x\to 7}\frac{\sqrt{x+9}-4}{x-7}$	9. $\lim_{x\to -2}(3x^2-x+1)$	10. $\lim_{x\to 3}(2x^2+5x-6)$
11. $\lim_{x\to -7}\frac{2x^3+11x^2-21x}{x^2+7x}$	12. $\lim_{x\to 8}\frac{x^2+2x-80}{x-8}$	13. $\lim_{x\to 0}\frac{\sqrt{x+9}-3}{x}$	14. $\lim_{x\to 0}\frac{\sqrt{x+11}-\sqrt{11}}{x}$	15. $\lim_{x\to 5}\sqrt{4x-9}$

16. $\lim_{x\to -1}\sqrt{3-x}$

17. $\lim_{h\to 0}\frac{\frac{1}{x+h}-\frac{1}{x}}{h}$

18. $\lim_{h\to 0}\frac{5\sqrt{x+h}-5\sqrt{x}}{h}$

19. $\lim_{x\to \frac{1}{3}}\frac{6x^2+13x-5}{3x-1}$

20. $\lim_{x\to 0}\frac{7x^2+x}{x}$

21. $\lim_{x\to 2}\frac{\sqrt{5x-6}}{x}$

22. $\lim_{x\to \frac{\pi}{2}}\tan\left(\frac{x}{2}\right)$

23. $\lim_{x\to 1} 3$

24. $\lim_{x\to -3} 14$

25. $\lim_{x\to 0}\frac{\frac{1}{x+3}-\frac{1}{3}}{x}$

26. $\lim_{x\to 0}\frac{\frac{1}{(x+2)^2}-\frac{1}{4}}{x}$

27. $\lim_{x\to 0}(-2)$

28. $\lim_{x\to 1}\frac{\sqrt{x+5}+\sqrt{6}}{x}$

29. $\lim_{x\to 0} \frac{x^2+2x-8}{x-4}$

30. $\lim_{x\to -2} \frac{x^2-4x-10}{x}$

31. $\lim_{x\to 0} \frac{3x^2+5x}{x}$

32. $\lim_{x\to 4} \frac{5x^2-21x+4}{x-4}$

33. $\lim_{x\to \frac{1}{2}} \frac{1-x-2x^2}{2x-1}$

34. $\lim_{x\to \pi} \cos x$

35. $\lim_{x\to \frac{\pi}{8}} \sin(4x)$

36. $\lim_{x\to 2} \frac{x^2+6x-16}{2-x}$

37. $\lim_{x\to 5} \frac{2x^2-17x+35}{5-x}$

38. $\lim_{x\to 0} \frac{(1-\cos^2 x)\sin x}{x^2}$

39. $\lim_{h\to 0} \frac{(x+h)^2+6(x+h)-(x^2+6x)}{h}$

40. $\lim_{h\to 0} \frac{4(x+h)^2-5(x+h)-2-(4x^2-5x-2)}{h}$

On the AP exam, there will be questions where you must find the hole of a function. This is basically finding the limit as you approach the discontinuity. The problems on the exam will be worded differently, but if you can recognize what to do, they are not that difficult.

The following functions have a removable discontinuity (hole). If we fill in this hole to make the function continuous, what is the coordinate point to fill in?

41. $\frac{x^2-x-12}{x-4}$

42. $\frac{x^2+7x}{2x}$

43. $\frac{2x-1}{2x^2+x-1}$

44. $\frac{3x^2+13x+4}{x+4}$

Using the following piecewise functions, find the given values.

$$g(x)=\begin{cases}\sqrt{5-x}, & x<-4\\ x^2-5, & -4\le x<2\\ x-3, & x\ge 2\end{cases}$$

$\lim_{x\to 2^-} g(x) =$

$\lim_{x\to -4^+} g(x) =$

$g(2) =$

$\lim_{x\to -4^-} g(x) =$

$\lim_{x\to 2^+} g(x) =$

$\lim_{x\to 2} g(x) =$

$\lim_{x\to -4} g(x) =$

$g(-4) =$

$$h(x)=\begin{cases}-|x|, & x\le -5\\ 20-x^2, & -5<x\le 3\\ 4x-1, & x>3\end{cases}$$

$\lim_{x\to -5^+} h(x) =$

$\lim_{x\to -5} h(x) =$

$h(3) =$

$\lim_{x\to -5^-} h(x) =$

$\lim_{x\to 3^+} h(x) =$

$\lim_{x\to 3} h(x) =$

$h(-5) =$

$\lim_{x\to 3^-} h(x) =$

$$w(\theta)=\begin{cases}\sin\theta, & \theta\le\pi\\ \cos\theta, & \pi<\theta<2\pi\\ \tan\theta, & \theta>2\pi\end{cases}$$

$\lim_{x\to \pi^-} w(\theta) =$

$w(\pi) =$

$\lim_{x\to \pi^+} w(\theta) =$

$\lim_{x\to 2\pi^-} w(\theta) =$

$\lim_{x\to \pi} w(\theta) =$

$\lim_{x\to 2\pi^+} w(\theta) =$

$\lim_{x\to 2\pi} w(\theta) =$

$w(2\pi) =$

1.2 Limits Analytically

Test Prep

1. $\lim_{x\to -1} \cos(\pi x) =$

(A) π (B) 1 (C) 0 (D) -1 (E) The limit does not exist

2. If $f(x) = \begin{cases} \ln 3x, & 0 < x \le 3 \\ x \ln 3, & 3 < x \le 4 \end{cases}$, then $\lim_{x \to 3} f(x)$ is

(A) $\ln 9$ (B) $\ln 27$ (C) $3 \ln 3$ (D) $3 + \ln 3$ (E) nonexistent

3. Evaluate $\lim_{x \to 1} \frac{\ln x}{3x}$ is

(A) 0 (B) $\frac{3}{e}$ (C) e (D) 3 (E) The limit does not exist.

4. $\lim_{x \to 0} 4 \frac{\sin x \cos x - \sin x}{x^2}$ is

(A) 2 (B) $\frac{40}{3}$ (C) ∞ (D) 0 (E) undefined

5. $\lim_{x \to a} \frac{x^2 - 2ax + a^2}{x - a} =$

(A) $-\infty$ (B) a (C) 0 (D) ∞ (E) The limit does not exist.

6. Let $f(x) = \begin{cases} \frac{\sin x}{x}, & x \ge 0 \\ \cos x, & x < 0 \end{cases}$ Which of the following statements about $f(x)$ is true?

I. $\lim_{x \to 0^+} f(x) = 1$

II. $\lim_{x \to 0^-} f(x) = 1$

III. $\lim_{x \to 0} f(x) = 1$

(A) None of these statements are true. (B) I only (C) II only (D) I and II only (E) I, II, and III

7. $\lim_{x \to 0} \left(\frac{3x^2 + 5\cos x - 5}{2x} \right) =$

(A) 0 (B) $\frac{5}{2}$ (C) 3 (D) 4 (E) Does not exist

8. If $f(x) = \begin{cases} \ln x & \text{for } 0 < x \le 2 \\ x^2 \ln 2 & \text{for } 2 < x \le 4, \end{cases}$ then $\lim_{x \to 2} f(x)$ is

(A) $\ln 2$ (B) $\ln 8$ (C) $\ln 16$ (D) 4 (E) nonexistent