odds = solutions evens = answers

1.2 Limits Analytically

Calculus

Name: Solutions

Practice

Evaluate each limit.

1. $\lim_{x\to 2}(x-x^2)$ $2-2^2$ $\boxed{-2}$	2. $\lim_{x\to 5}(x+1)^2$ $\boxed{36}$	3. $\lim_{x\to 1}\frac{x^2-5x}{x-1} = \frac{x(x-5)}{(x-1)}$ $\frac{1(1-5)}{1-1} = \frac{-4}{0}$ $\boxed{DNE}$	4. $\lim_{x\to 1}\frac{x^2+x-30}{x-1}$ $\boxed{DNE}$	5. $\lim_{x\to 0}\frac{3x}{\sin x}$ $\lim_{x\to 0} 3\cdot\frac{x}{\sin x}$ $3\cdot 1$ $\boxed{3}$
6. $\lim_{x\to 0}\frac{\sin(2x)}{3x}$ $\boxed{\frac{2}{3}}$	7. $\lim_{x\to 0}\frac{\sqrt{x+7}-\sqrt{7}}{x}\cdot\frac{\sqrt{x+7}+\sqrt{7}}{\sqrt{x+7}+\sqrt{7}}$ $\lim_{x\to 0}\frac{(x+7)-(7)}{x(\sqrt{x+7}+\sqrt{7})}$ $\lim_{x\to 0}\frac{x}{x(\sqrt{x+7}+\sqrt{7})}$ $=\frac{1}{\sqrt{7}+\sqrt{7}} = \boxed{\frac{1}{2\sqrt{7}}}$	8. $\lim_{x\to 7}\frac{\sqrt{x+9}-4}{x-7}$ $\boxed{\frac{1}{8}}$	9. $\lim_{x\to -2}(3x^2-x+1)$ $3(-2)^2-(-2)+1$ $12+3$ $\boxed{15}$	10. $\lim_{x\to 3}(2x^2+5x-6)$ $\boxed{27}$
11. $\lim_{x\to -7}\frac{2x^3+11x^2-21x}{x^2+7x}$ $\lim_{x\to -7}\frac{\cancel{x}(2x-3)\cancel{(x+7)}}{\cancel{x}\cancel{(x+7)}}$ $2(-7)-3$ $\boxed{-17}$	12. $\lim_{x\to 8}\frac{x^2+2x-80}{x-8}$ $\boxed{18}$	13. $\lim_{x\to 0}\frac{\sqrt{x+9}-3}{x}\cdot\frac{\sqrt{x+9}+3}{\sqrt{x+9}+3}$ $\lim_{x\to 0}\frac{(x+9)-9}{x(\sqrt{x+9}+3)}$ $\lim_{x\to 0}\frac{1}{\sqrt{x+9}+3}$ $\boxed{\frac{1}{6}}$	14. $\lim_{x\to 0}\frac{\sqrt{x+11}-\sqrt{11}}{x}$ $\boxed{\frac{1}{2\sqrt{11}}}$	15. $\lim_{x\to 5}\sqrt{4x-9}$ $\sqrt{4(5)-9}$ $\sqrt{20-9}$ $\boxed{\sqrt{11}}$

16. $\lim_{x\to -1} \sqrt{3-x}$

$\boxed{2}$

17. $\lim_{h\to 0} \frac{\frac{1}{x+h}-\frac{1}{x}}{h}$ $\cdot \frac{(x+h)}{(x+h)}$

$\lim_{h\to 0} \frac{\frac{x-(x+h)}{x(x+h)}}{h}$

$\lim_{h\to 0} \frac{-h}{x(x+h)} \cdot \frac{1}{h}$

$\frac{-1}{x(x+0)} = \boxed{-\frac{1}{x^2}}$

18. $\lim_{h\to 0} \frac{5\sqrt{x+h}-5\sqrt{x}}{h}$

$\boxed{\frac{5}{2\sqrt{x}}}$

19. $\lim_{x\to \frac{1}{3}} \frac{6x^2+13x-5}{3x-1}$

$\lim_{x\to \frac{1}{3}} \frac{(3x-1)(2x+5)}{3x-1}$

$2\left(\frac{1}{3}\right)+5$

$\frac{2}{3}+\frac{15}{3}$

$\boxed{\frac{17}{3}}$

20. $\lim_{x\to 0} \frac{7x^2+x}{x}$

$\boxed{1}$

21. $\lim_{x\to 2} \frac{\sqrt{5x-6}}{x}$

$\frac{\sqrt{5(2)-6}}{2}$

$\frac{\sqrt{4}}{2} = \boxed{1}$

22. $\lim_{x\to \frac{\pi}{2}} \tan\left(\frac{x}{2}\right)$

$\boxed{1}$

23. $\lim_{x\to 1} 3$

$\boxed{3}$

24. $\lim_{x\to -3} 14$

$\boxed{14}$

25. $\lim_{x\to 0} \frac{\frac{1}{x+3}-\frac{1}{3}}{x}$ $\cdot \frac{x+3}{x+3}$

$\lim_{x\to 0} \frac{\frac{3-(x+3)}{3(x+3)}}{x}$

$\lim_{x\to 0} \frac{-x}{3(x+3)} \cdot \frac{1}{x}$

$\lim_{x\to 0} \frac{-1}{3(x+3)}$

$\boxed{-\frac{1}{9}}$

26. $\lim_{x\to 0} \frac{\frac{1}{(x+2)^2}-\frac{1}{4}}{x}$

$\boxed{-\frac{1}{4}}$

27. $\lim_{x\to 0}(-2)$

no work needed

$\boxed{-2}$

28. $\lim_{x\to 1} \frac{\sqrt{x+5}+\sqrt{6}}{x}$

$\boxed{2\sqrt{6}}$

29. $\lim_{x\to 0} \frac{x^2+2x-8}{x-4}$

$\frac{(0)^2+2(0)-8}{(0)-4}$

$\boxed{2}$

30. $\lim_{x\to -2} \frac{x^2-4x-10}{x}$

$\boxed{-1}$

31. $\lim_{x\to 0} \frac{3x^2+5x}{x} = \frac{x(3x+5)}{x}$

$\lim_{x\to 0}(3x+5)$

$\boxed{5}$

32. $\lim_{x\to 4} \frac{5x^2-21x+4}{x-4}$

$\boxed{19}$

33. $\lim_{x\to \frac{1}{2}} \frac{1-x-2x^2}{2x-1}$

$\lim_{x\to \frac{1}{2}} -\frac{(2x-1)(x+1)}{2x-1}$

$-\left(\frac{1}{2}+1\right)$

$\boxed{-\frac{3}{2}}$

34. $\lim_{x\to \pi} \cos x$

$\boxed{-1}$

35. $\lim_{x\to \frac{\pi}{8}} \sin(4x)$

$\sin\left(\frac{\pi}{2}\right)$

$\boxed{1}$

36. $\lim_{x\to 2} \frac{x^2+6x-16}{2-x}$

$\boxed{-10}$

37. $\lim_{x\to 5} \frac{2x^2-17x+35}{5-x}$

$\lim_{x\to 5} \frac{(x-5)(2x-7)}{-(x-5)}$

$\frac{2(5)-7}{-1}$

$\boxed{-3}$

38. $\lim_{x\to 0} \frac{(1-\cos^2 x)\sin x}{x^2}$

$\frac{(1-\cos x)}{x} \cdot \frac{(1+\cos x)\sin x}{x}$

$(0)(1+\cos x)(1)$

$\boxed{0}$

39. $\lim_{h\to 0} \frac{(x+h)^2+6(x+h)-(x^2+6x)}{h}$

$\lim_{h\to 0} \frac{x^2+2hx+h^2+6x+6h-x^2-6x}{h}$

$\lim_{h\to 0} \frac{2hx+h^2+6h}{h}$

$\lim_{h\to 0}(2x+h+6)$

$\boxed{2x+6}$

40. $\lim_{h\to 0} \frac{4(x+h)^2-5(x+h)-2-(4x^2-5x-2)}{h}$

$\boxed{8x-5}$

On the AP exam, there will be questions where you must find the hole of a function. This is basically finding the limit as you approach the discontinuity. The problems on the exam will be worded differently, but if you can recognize what to do, they are not that difficult.

The following functions have a removable discontinuity (hole). If we fill in this hole to make the function continuous, what is the coordinate point to fill in?

41. $\frac{x^2-x-12}{x-4}$ $\frac{(x+3)\cancel{(x-4)}}{\cancel{x-4}}$ Disc. at $x=4$ $x+3$ $(4)+3=7$ $\boxed{\text{hole @ }(4,7)}$	42. $\frac{x^2+7x}{2x}$ $\boxed{\text{hole @ }(0,\frac{7}{2})}$	43. $\frac{2x-1}{2x^2+x-1}$ $\frac{\cancel{2x-1}}{\cancel{(2x-1)}(x+1)}$ Disc. at $x=\frac{1}{2}$ $\frac{1}{x+1} \rightarrow \frac{1}{\frac{1}{2}+1}=\frac{1}{\frac{3}{2}}$ $\boxed{\text{hole @ }(\frac{1}{2},\frac{2}{3})}$	44. $\frac{3x^2+13x+4}{x+4}$ $\boxed{\text{hole @ }(-4,-11)}$

Using the following piecewise functions, find the given values.

$g(x)=\begin{cases}\sqrt{5-x}, & x<-4\\ x^2-5, & -4\le x<2\\ x-3, & x\ge 2\end{cases}$	$h(x)=\begin{cases}-\lvert x\rvert, & x\le -5\\ 20-x^2, & -5<x\le 3\\ 4x-1, & x>3\end{cases}$	$w(\theta)=\begin{cases}\sin\theta, & \theta\le\pi\\ \cos\theta, & \pi<\theta<2\pi\\ \tan\theta, & \theta>2\pi\end{cases}$
$\lim_{x\to 2^-} g(x) = -1$	$\lim_{x\to -5^+} h(x) = -5$	$\lim_{x\to \pi^-} w(\theta) = 0$
$\lim_{x\to -4^+} g(x) = 11$	$\lim_{x\to -5} h(x) = -5$	$w(\pi) = 0$
$g(2) = -1$	$h(3) = 11$	$\lim_{x\to \pi^+} w(\theta) = -1$
$\lim_{x\to -4^-} g(x) = 3$	$\lim_{x\to -5^-} h(x) = -5$	$\lim_{x\to 2\pi^-} w(\theta) = 1$
$\lim_{x\to 2^+} g(x) = -1$	$\lim_{x\to 3^+} h(x) = 11$	$\lim_{x\to \pi} w(\theta) =$ DNE
$\lim_{x\to 2} g(x) = -1$	$\lim_{x\to 3} h(x) = 11$	$\lim_{x\to 2\pi^+} w(\theta) = 0$
$\lim_{x\to -4} g(x) =$ DNE	$h(-5) = -5$	$\lim_{x\to 2\pi} w(\theta) =$ DNE
$g(-4) = 11$	$\lim_{x\to 3^-} h(x) = 11$	$w(2\pi) =$ DNE

Test Prep: 1D, 2E, 3A, 4D, 5C, 6E, 7A, 8E