Calculus

Write your questions and thoughts here!

1.3 Asymptotes

Name:__________________________

Notes

Recall: Vertical Asymptote   Horizontal Asymptote

Vertical Asymptotes:

True or False. If you have the function $f(x) = \frac{blah,\ blah,\ blah}{x-a}$ then there must be a vertical asymptote at $x = a$.

Use the function $f(x) = \frac{x^2+2x-8}{x^2+x-12}$ to answer the following.

1. Identify all vertical asymptotes.
2. Evaluate $\lim_{x\to 3^-} f(x)$
3. Evaluate $\lim_{x\to 3^+} f(x)$

Horizontal Asymptotes: (End-behavior)

What does the y-value approach as the x-value approaches negative infinity AND positive infinity? Does it approach a specific number, or is it growing without bound?

Basic Rules for Horizontal Asymptotes:

________________ grows faster means $\frac{\text{not as big}}{\text{super duper BIG number!}} = 0$

If the numerator and denominator grow __________ fast, then you have $\frac{\text{BIG number!}}{\text{BIG number!}} = 1$

If the ____________ grows faster than the denominator, then you have $\frac{\text{BIG number!}}{\text{not as big}} = \infty$

First, you need to recognize which functions grow faster as x-values get larger and larger.

Rank Fastest to Slowest	$f(x)$	$x = 1$	$x = 10$	$x = 100$	$x = 1000$
	x^2				
	x^3				
	x^{10}				
	2^x				
	e^x				
	4^x				
	$\ln x$				

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Find the horizontal asymptote(s) of each function.

4. $y = \frac{x^2+4}{3x-5}$	5. $y = \frac{x+4}{3x-5}$	6. $y = \frac{x+4}{3x^2-5}$

What about weird ones like this: $y = \frac{\sqrt{x^2+2}}{x-1}$

Evaluate the limit.

7. $\lim_{x\to\infty} \frac{\sqrt{4x^2+x-2}}{3x-1}$	8. $\lim_{x\to-\infty} \frac{\sqrt{4x^2+x-2}}{3x-1}$	9. $\lim_{x\to\infty} -4e^{\frac{1}{x}}$	10. $\lim_{x\to\infty} 5e^{-x}$

<u>Trig Function's Horizontal Asymptotes:</u>

Evaluate the limit.

11. $\lim_{x\to-\infty} \frac{\sin x}{x}$	12. $\lim_{x\to\infty} -3\cos\frac{1}{x}$	13. $\lim_{x\to\infty} \sin x$	14. $\lim_{x\to\infty} 5x\cos x$

<u>Squeeze Theorem:</u> a.k.a. "Sandwich Theorem" or "Pinching Theorem"

If $(x) \leq \quad (x) \leq \quad (x)$

and if $\lim f(x) =$ and $\lim h(x) =$

then $\lim g(x) =$

Use the Squeeze Theorem to evaluate the limit.

15. $\lim_{x\to 0} x^2 \cos\left(\frac{1}{x^2}\right)$

Now summarize what you learned!

1.3 Asymptotes

Calculus Name: ____________________________ **Practice**

Identify all vertical asymptotes of each function.

1. $f(x) = \frac{x^2-x-12}{x+7}$

2. $f(x) = \frac{x^3+4x^2-24x}{x^2-1x}$

3. $f(x) = \frac{7x^2+4x-3}{7x-3}$

4. $f(x) = \frac{3x^2-11x+10}{x-2}$

Identify all horizontal asymptotes of each function.

5. $f(x) = \frac{\sqrt{25x^4+2x}}{x^2}$

6. $f(x) = \frac{\sqrt{7x^6+3x^2+x}}{x^3+4x^2}$

7. $f(x) = \frac{\sqrt{9x^8-2x^3-6x}}{2x^4-10x} + 3$

8. $f(x) = \frac{3x^2}{\sqrt{3x^4-2x}}$

Using the Squeeze Theorem, evaluate each limit. SHOW WORK!

9. $\lim_{x\to 0} x\cos\left(\frac{1}{x}\right)$

10. $\lim_{x\to 0} x^2\sin\left(\frac{1}{x^2}\right)$

11. $\lim_{x\to 0} x\sin\left(\frac{1}{x^2}\right)$

Evaluate each limit.

12. $\lim_{x\to\infty} \frac{-x+2}{x^2+2x+2}$

13. $\lim_{x\to\infty} \left(\sin\frac{1}{x} - \frac{6x^2+2x}{3x^2}\right)$

14. $\lim_{x\to\infty} \left(5\cos\frac{1}{x}\right)$

15. $\lim_{x\to\infty} \frac{x^7}{4^x} - 5$

16. $\lim_{x\to\infty} 3^{-x} + 2$

17. $\lim_{x\to\infty} -3x\cos x$

18. $\lim_{x\to\infty} 2x\sin x$

19. $\lim_{x\to\infty} \frac{9x^4+4x^3+3}{x^7+2x^4+2x^3}$

20. $\lim_{x\to-\infty} \frac{3x^2-5x+11}{x^2-2x}$

21. $\lim_{x\to\infty} \cos\left(\frac{2x-\pi x^2}{x^2}\right)$

22. $\lim_{x\to\infty} \left(\frac{\sin x}{x} - 4\right)$

23. $\lim_{x\to\infty} \frac{-x^4-3x^2-8}{5x^4+7x+13}$

24. $\lim_{x\to\infty} \frac{x^3-7x^2+8}{x^2+7x-2}$

25. $\lim_{x\to\infty} x^2 2^{-x}$

26. $\lim_{x\to\infty} \frac{e^7}{9^x}$

27. $\lim_{x\to-\infty} \frac{3x^2-5x^7+6}{x^7-15x^4}$

28. $\lim_{x\to\infty} \frac{2x^4+3x^2+10}{5x^2+6x-1}$

29. $\lim_{x\to\infty} \left(\frac{\sin x}{x} + 2\right)$

30. $\lim_{x\to\infty} \cos\left(\frac{x^5}{e^x}\right) + 4$

31. $\lim_{x\to\infty} \frac{3x^6-5x^3+6}{x^3+x^8-2x^4}$

32. $\lim_{x\to\infty} \sin(2x)$

33. $\lim_{x\to\infty} \cos\left(\frac{\pi x^2+\frac{\sqrt{2}}{2}x}{5-2x^2}\right)$

34. $\lim_{x\to\infty} \cos\left(\frac{\frac{\sqrt{2}}{2}x-\pi x^2}{x^2-x^3+2}\right)$

1.3 Asymptotes

Test Prep

1. $\lim_{x\to 0^+} \frac{\cos x}{x} =$

(A) $-\infty$ (B) -1 (C) 0 (D) 1 (E) ∞

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2. Which of the following functions grows the fastest?

(A) $a(u) = \left(\frac{1}{2}\right)^u$ (B) $b(u) = u^{100} + u^{99}$ (C) $c(u) = 4^u$

(D) $d(u) = 200e^u$ (E) $e(u) = 3^u + u^3$

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3. Consider the functions $f(x) = \frac{1}{x}, x \neq 0$, and $g(x) = x\sin\frac{1}{x}, x \neq 0$. Which of the following describes the behavior of f and g as $x \to 0$?

(A) $\lim_{x\to 0} f(x) = 0$ and $\lim_{x\to 0} g(x) = 0$

(B) $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$ do not exist.

(C) $\lim_{x\to 0} f(x) = 0$ and $\lim_{x\to 0} g(x)$ does not exist.

(D) $\lim_{x\to 0} f(x)$ does not exist and $\lim_{x\to 0} g(x) = 0$

(E) $\lim_{x\to 0} f(x) = \infty$ and $\lim_{x\to 0} g(x) = 0$

4. Suppose that $g(x) = \sin^2 x\sqrt{x^6+4}$, and $\lim_{x\to\infty}\frac{g(x)}{f(x)} = 0$. Which of the following functions could be f?

(A) x (B) x^2 (C) x^3 (D) x^4 (E) $\ln x$

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5. Which of the following statements are true for the function $f(x) = \frac{2x^3+3x+1}{2^x}$

I. $f(x)$ has a horizontal asymptote of $y = 1$
II. $f(x)$ has a horizontal asymptote of $y = 0$
III. $f(x)$ has a vertical asymptote of $x = 0$

(A) I only (B) II only (C) III only (D) I and III only (E) II and III only

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6. Which of the following functions has both a vertical and horizontal asymptote?

(A) $f(x) = \frac{1}{1+e^{-x}}$ (B) $f(x) = \tan x$ (C) $f(x) = \frac{x}{x^2+2}$

(D) $f(x) = \frac{x}{x^2-2}$ (E) $f(x) = \frac{x^2+2}{x}$

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7. The function $f(x) = \begin{cases} \frac{x^2+2x+3}{x^2-1}, & x \geq 0 \\ \frac{x}{e^x}, & x < 0 \end{cases}$ has which of the following asymptotes?

(A) $y = 0$ only. (B) $y = 1$ only. (C) $y = 1,\ x = 1$ only.

(D) $y = 1,\ x = \pm 1$ only. (E) $y = 0,\ y = 1,\ x = \pm 1$.

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8. If the function $f(x) = \frac{-ax^3+bx^2+cx+d}{e^{-x}-wx^3+w}$ has a horizontal asymptote of $y = 2$ and a vertical asymptote of $x = 0$, then $w - a =$

(A) -1 (B) 0 (C) 1 (D) ∞ (E) The limit does not exist.

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9. What are all horizontal asymptotes of the graph of $y = \frac{5+2^x}{1-2^x}$ in the xy-plane?

(A) $y = -1$ only (B) $y = 0$ only (C) $y = 5$ only

(D) $y = -1$ and $y = 0$ (E) $y = -1$ and $y = 5$