

### 1.3 Asymptotes

Calculus

Name: Solutions

**Practice**

Identify all vertical asymptotes of each function.

1.  $f(x) = \frac{x^2-x-12}{x+7}$   
 $\frac{(x+3)(x-4)}{x+7}$

V.A. at  $x=-7$

2.  $f(x) = \frac{x^3+4x^2-24x}{x^2-1x}$

V.A. at  $x=1$

3.  $f(x) = \frac{7x^2+4x-3}{7x-3}$   
 $\frac{(7x-3)(x+1)}{7x-3}$

No vertical asymptote

4.  $f(x) = \frac{3x^2-11x+10}{x-2}$

No vertical asymptote

Identify all horizontal asymptotes of each function.

5.  $f(x) = \frac{\sqrt{25x^4+2x}}{x^2}$   
 $\frac{5x^2 + \text{blah}}{x^2}$   
 $x \rightarrow \infty$  and  $x \rightarrow -\infty$   
 $y=5$

6.  $f(x) = \frac{\sqrt{7x^6+3x^2+x}}{x^2+4x^2}$   
 $y=\sqrt{7}$  and  $y=-\sqrt{7}$

7.  $f(x) = \frac{\sqrt{9x^8-2x^3-6x}}{2x^4-10x} + 3$   
 $\frac{3x^4 + \text{blah, blah}}{2x^4 + \text{blah}} + 3$   
 $\frac{3}{2} + 3$   
 $y=\frac{9}{2}$

8.  $f(x) = \frac{3x^2}{\sqrt{3x^4-2x}}$   
 $y=\sqrt{3}$

Using the Squeeze Theorem, evaluate each limit. SHOW WORK!

9.  $\lim_{x \rightarrow 0} x \cos\left(\frac{1}{x}\right)$   
 $-1 \leq \cos\left(\frac{1}{x}\right) \leq 1$   
 $-x \leq x \cos\left(\frac{1}{x}\right) \leq x$   
 Take the limit:  
 $0 \leq \lim_{x \rightarrow 0} x \cos\left(\frac{1}{x}\right) \leq 0$   
 $0$

10.  $\lim_{x \rightarrow 0} x^2 \sin\left(\frac{1}{x^2}\right)$   
 $0$

11.  $\lim_{x \rightarrow 0} x \sin\left(\frac{1}{x}\right)$   
 $-1 \leq \sin\left(\frac{1}{x}\right) \leq 1$   
 $-x \leq x \sin\left(\frac{1}{x}\right) \leq x$   
 Take the limit:  
 $0 \leq \lim_{x \rightarrow 0} x \sin\left(\frac{1}{x}\right) \leq 0$   
 $0$

Evaluate each limit.

12.  $\lim_{x \rightarrow \infty} \frac{-x+2}{x^2+2x+2}$   
 $0$

13.  $\lim_{x \rightarrow \infty} \left(\sin \frac{1}{x} - \frac{6x^2+2x}{3x^2}\right)$   
 $\sin(0) - 2$   
 $0 - 2$   
 $-2$

14.  $\lim_{x \rightarrow \infty} \left(5 \cos \frac{1}{x}\right)$   
 $5$

15.  $\lim_{x \rightarrow \infty} \frac{x^7}{4^x} - 5$   
 big MUCH BIGGER  
 $0 - 5$   
 $-5$

16.  $\lim_{x \rightarrow \infty} 3^{-x} + 2$   
 $2$

17.  $\lim_{x \rightarrow \infty} -3x \cos x$   
 Oscillating  
 Does not exist  
 means "therefore"

18.  $\lim_{x \rightarrow \infty} 2x \sin x$   
 Does not exist

19.  $\lim_{x \rightarrow \infty} \frac{9x^4+4x^3+3}{x^7+2x^4+2x^3}$   
 big MUCH BIGGER  
 $0$

20.  $\lim_{x \rightarrow \infty} \frac{3x^2-5x+11}{x^2-2x}$   
 $3$

21.  $\lim_{x \rightarrow \infty} \cos\left(\frac{2x-\pi x^2}{x^2}\right)$   
 $\cos(-\pi)$   
 $-1$

22.  $\lim_{x \rightarrow \infty} \left(\frac{\sin x}{x} - 4\right)$   
 $-4$

23.  $\lim_{x \rightarrow \infty} \frac{-x^3-3x^2-8}{5x^3+7x+13}$   
 same degree  
 $-\frac{1}{5}$

24.  $\lim_{x \rightarrow \infty} \frac{x^2-7x^2+8}{x^2+7x-2}$   
 $\infty$

25.  $\lim_{x \rightarrow \infty} x^2 2^{-x}$   
 $\lim_{x \rightarrow \infty} \frac{x^2}{2^x}$   
 big MUCH BIGGER =  $0$

26.  $\lim_{x \rightarrow \infty} \frac{e^7}{9^x}$   
 $0$

27.  $\lim_{x \rightarrow \infty} \frac{3x^2-5x+6}{x^2-15x^4}$   
 same degree  
 $-5$

$$28. \lim_{x \rightarrow \infty} \frac{2x^4 + 3x^2 + 10}{5x^2 + 6x - 1}$$

$\infty$

$$29. \lim_{x \rightarrow \infty} \left( \frac{\sin x}{x} + 2 \right)$$

$0 + 2$

2

$$30. \lim_{x \rightarrow \infty} \cos\left(\frac{x^5}{e^x}\right) + 4$$

5

$$31. \lim_{x \rightarrow \infty} \frac{3x^6 - 5x^2 + 6}{x^2 + x^8 - 2x^4}$$

*big  
MUCH BIGGER*

0

$$32. \lim_{x \rightarrow \infty} \sin(2x)$$

Does not exist

$$33. \lim_{x \rightarrow \infty} \cos\left(\frac{\pi x^2 + \frac{\sqrt{2}}{2}x}{5 - 2x^2}\right)$$

*same degree*

0

$\cos\left(-\frac{\pi}{2}\right)$

$$34. \lim_{x \rightarrow \infty} \cos\left(\frac{\frac{\sqrt{2}}{2}x - \pi x^2}{x^2 - x^2 + 2}\right)$$

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Test Prep: 1E, 2C, 3D, 4D, 5B, 6D, 7C, 8C, 9E