1.4 Continuity

Calculus

Name: Solutions

Practice

Identify and classify each point of discontinuity of the given function.

1. $f(x) = \frac{x}{x+1}$ $=0$

V.A. at $x=-1$

2. $f(x) = \frac{x^2}{x^2+3x}$

hole at $x=0$

V.A. at $x=-3$

3. $f(x) = \frac{2x}{2x-5}$

V.A. at $x = 5/2$

4. $f(x) = \sqrt{2-6x}$

Continuous on its domain.

5. $f(x) = \frac{x+2}{x^2-2x-8}$

$(x-4)(x+2)$

hole at $x=-2$

V.A. at $x=4$

6. $f(x) = \frac{4x+5}{3}$

Continuous function

7. $f(x) = \begin{cases} 3-2x, & x<2 \\ x-3, & x \geq 2 \end{cases}$

$3-2(2) = -1$

$(2)-3 = -1$

Continuous function

8. $f(x) = \begin{cases} 5x+1, & x \leq -1 \\ x+3, & x > -1 \end{cases}$

Jump Disc. at $x=-1$

9. $f(x) = \begin{cases} \frac{x^2-1}{x+11}, & x<4 \\ x-3, & x>4 \\ 5, & x=4 \end{cases}$

$\frac{4^2-1}{4+11} = \frac{15}{15} = 1$

$4-3=1$

$x+11 \neq 0$

hole at $x=4$

V.A. at $x=-11$

10. $f(x) = \begin{cases} \frac{x}{e}+3, & x<e \\ \ln x^4, & x \geq e \end{cases}$

Continuous function

Find the domain of each function.

11. $s(x) = \frac{\sqrt{6x-2}}{5}$ ≥ 0

$6x \geq 2$

$x \geq 1/3$

12. $w(t) = \frac{6}{\sqrt{2t+10}}$

$t > -5$

13. $f(x) = \frac{x}{\sqrt{6-2x}}$ > 0

$-2x > -6$

$x < 3$

14. $v(t) = \frac{3t}{t\sqrt{t+7}}$

$t > -7, \ t \neq 0$

15. $g(x) = \frac{x+1}{x^2+5x+4}$

$(x+1)(x+4) = 0$

$\mathbb{R}, x \neq -1, x \neq -4$

16. $g(w) = \frac{2}{4-\sqrt{w}}$

$w \geq 0, \ w \neq 16$

17. $s(t) = \sqrt[3]{t-8}$	18. $h(t) = \frac{\sqrt{4-t}}{t-5}$	19. $g(x) = x^2 + 11x + 30$
$\mathbb{R}$ [All real numbers]	$t \leq 4$	$\mathbb{R}$ [All real numbers]

Below is a table of values for a continuous function f. Use this table to answer questions 20-22.

x	3	4	5	6	7
$f(x)$	4	1	-3	-1	6

20. On the interval $3 \leq x \leq 7$, must there be a value of x for which $f(x) = 5$? Explain.

Yes. On the interval $6 \leq x \leq 7$, the function changes from -1 to 6. It must equal 5 at some point in that interval by way of the Intermediate Value Theorem.

21. On the interval $3 \leq x \leq 7$, ***could*** there be a value of x for which $f(x) = 7$? Explain.

Yes. The function **COULD** increase to a value of 7 on any of the intervals, but it is not guaranteed because the largest value given is 6.

22. What is the minimum number of zeros f must have on the interval $3 \leq x \leq 7$? 2

Below is a table of values for a continuous function g. Use this table to answer questions 23-26.

x	0	2	15	32	50
$g(x)$	-1	10	17	-10	8

23. On the interval $0 \leq x \leq 15$, must there be a value of x for which $g(x) = -3$? Explain.

No. The lowest value of g from the table on the interval $0 \leq x \leq 15$ is -1. It is possible the value dips to $g(x) = -3$, but the IVT does not guarantee it.

24. On the interval $0 \leq x \leq 50$, must there be a value of x for which $g(x) = 11$? Explain.

Yes. On the interval $2 \leq x \leq 15$, the function changes from 10 to 17 and on the interval $15 \leq x \leq 32$, the function changes from 17 to -10. g must equal 11 at some point in those intervals by way of the Intermediate Value Theorem.

25. What is the minimum number of zeros g must have on the interval $15 \leq x \leq 50$? 2

26. What is the minimum number of zeros g must have on the interval $0 \leq x \leq 50$? 3

Test Prep: 1E, 2A, 3D, 4A, 5E, 6B, 7E, 8C, 9D

Grading the Free Response for 1.4 Continuity

DO NOT USE THIS UNTIL YOU ARE FINISHED WORKING ON THE PROBLEM!

Part a is worth 3 points.

1 Point: At $t = 1$, the particle is at $y = -2$.
1 Point: As t approaches 3, the particle moves in a positive direction along the y-axis.
1 Point: At $t = 3$, the particle disappears and reappears a $y = 3$.

Part b is worth 2 points.

1 Point: The particle is moving in a negative direction.
1 Point: The particle moves ever more slowly towards $y = 1$, but never reaches it.

Part c is worth 2 points.

1 Point: The Intermediate Value Theorem does not apply to either interval because f is not continuous on those intervals.
1 Point: Discontinuities at $t = 1$ and $t = 3$.