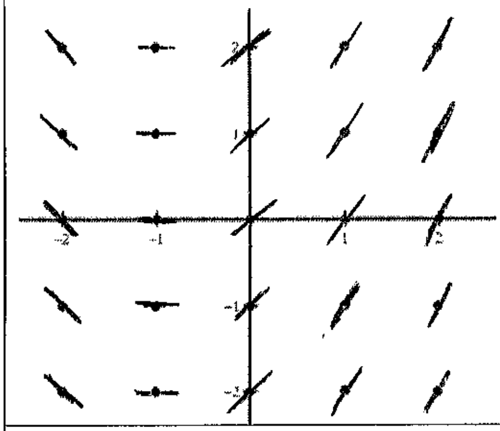
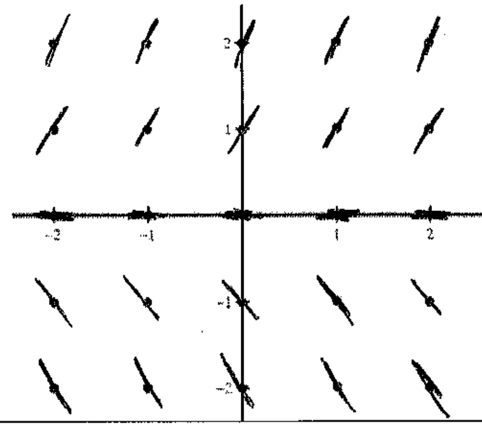
Draw a slope field for each of the following differential equations.

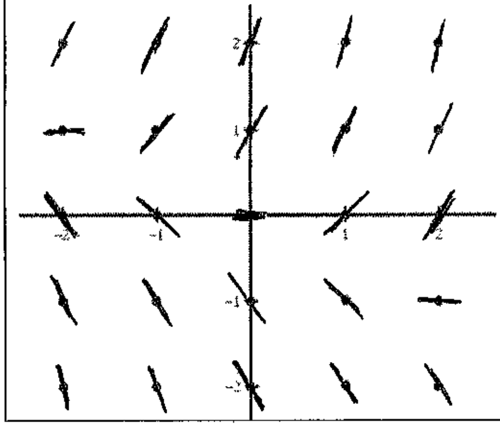
1. $\frac{dy}{dx} = x + 1$



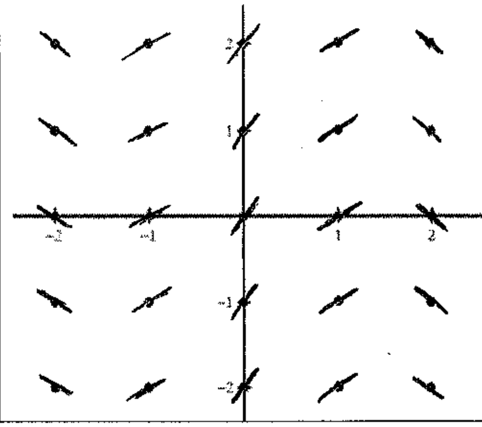
2. $\frac{dy}{dx} = 2y$



3. $\frac{dy}{dx} = x + 2y$

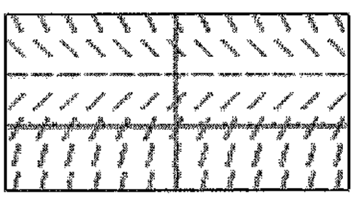


4. $\frac{dy}{dx} = \cos x$ calculator

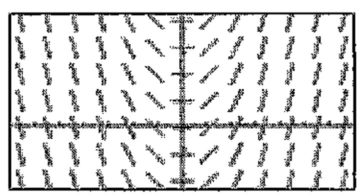


Match the slope fields with their differential equations.

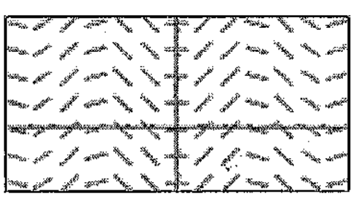
(A)



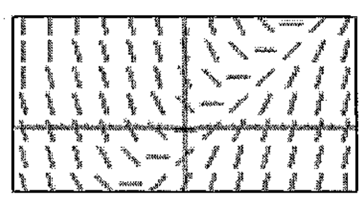
(B)



(C)



(D)



7. $\frac{dy}{dx} = \sin x$ C

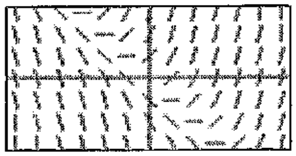
8. $\frac{dy}{dx} = x - y$ D

9. $\frac{dy}{dx} = 2 - y$ A

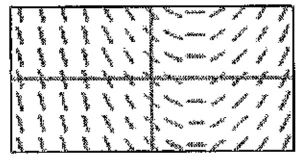
10. $\frac{dy}{dx} = x$ B

Match the slope fields with their differential equations.

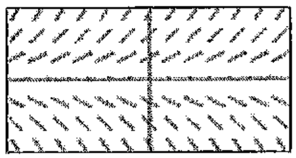
(A)



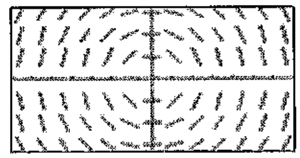
(B)



(C)



(D)



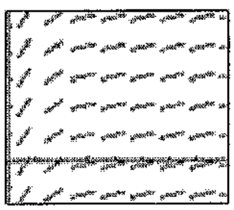
11. $\frac{dy}{dx} = .5x - 1$ B

12. $\frac{dy}{dx} = .5y$ C

13. $\frac{dy}{dx} = -\frac{x}{y}$ D

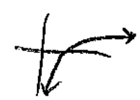
14. $\frac{dy}{dx} = x + y$ A

15.

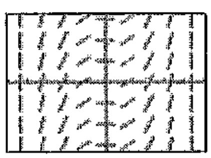


The slope field from a certain differential equation is shown above. Which of the following could be a specific solution to that differential equation?

(A) $y = x^2$ (B) $y = e^x$ (C) $y = e^{-x}$ (D) $y = \cos x$ (E) $y = \ln x$



16.



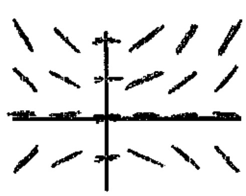
The slope field for a certain differential equation is shown above. Which of the following could be a specific solution to that differential equation?

(A) $y = \sin x$ (B) $y = \cos x$ (C) $y = x^2$ (D) $y = \frac{1}{6}x^3$ (E) $y = \ln x$

---

17. Consider the differential equation given by $\frac{dy}{dx} = \frac{xy}{2}$.

(a) On the axes provided, sketch a slope field for the given differential equation.



$y' = \frac{xy}{2} = \frac{(1)(1)}{2} = \frac{1}{2}$

$\boxed{y - 1 = \frac{1}{2}(x-1)}$

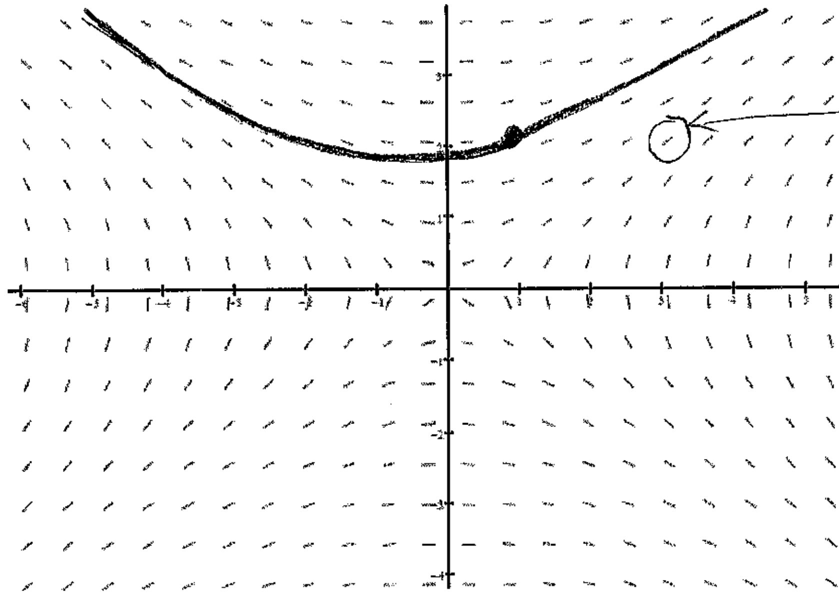
(b) Let f be the function that satisfies the given differential equation. Write an equation for the tangent line to the curve $y = f(x)$ through the point (1, 1). Then use your tangent line equation to estimate the value of $f(1.2)$.

$\boxed{f(1.2) \approx 1.1}$

$y = \frac{1}{2}(x-1) + 1$

$f(1.2) \approx \frac{1}{2}(1.2-1) + 1 = 1.1$

18. The figure below shows the slope field for the differential equation $\frac{dy}{dx} = \frac{x}{2y}$



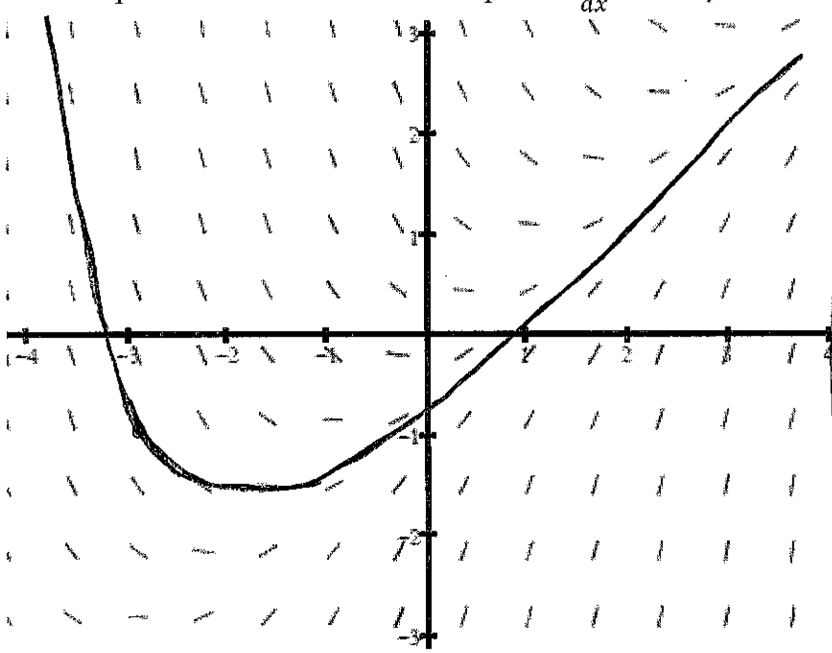
$m = \frac{3}{4}$

a) Calculate $\frac{dy}{dx}$ at the point (3,2) and verify that the result agrees with the figure.

$\frac{dy}{dx} = \frac{3}{2(2)} = \frac{3}{4}$

b) Sketch the graph of the particular solution of the differential equation that contains the point (1,2).

19. The figure below shows the slope field for the differential equation $\frac{dy}{dx} = x - y$



So...

$f'(1,1) = 0$

$f''(1,1) = 1-1+1 = 1$

f'' is positive which means concave up

so (1,1) is min

a) State a point where $\frac{dy}{dx} = 0$. Find $\frac{d^2y}{dx^2}$ and use it to verify if your point is a max or min.

example $(1,1)$

$\frac{dy}{dx} = 1 - 1 = 0$

$y' = (x-y)$

$y'' = 1 - y'$

$y'' = 1-(x-y)$

if you like this notation →

$\frac{dy}{dx} = x - y$

$\frac{d^2y}{dx^2} = 1 - \frac{dy}{dx}$

$\frac{d^2y}{dx^2} = 1-(x-y)$

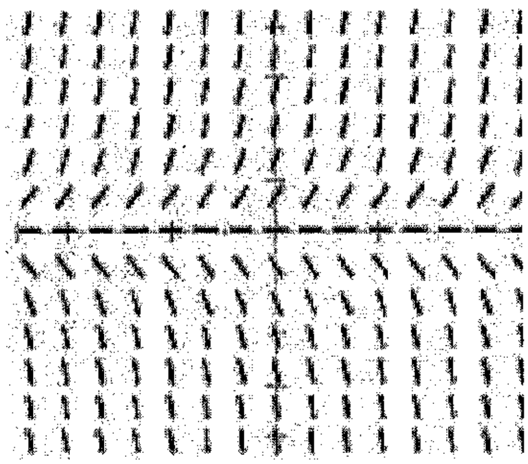
$\boxed{\frac{d^2y}{dx^2} = 1 - x + y}$

b) Sketch the graph of the particular solution of the differential equation that contains the point (−3,−1).

$y'' = 1 - x + y$

(above)

MULTIPLE CHOICE



1. D

The slope field for a differential equation is shown at right. Which statement is true for all solutions of the differential equation?

I. For $x < 0$, all solutions are decreasing. ✗
II. All solutions level off near the x-axis. ✓
III. For $y > 0$, all solutions are increasing ✓

(A) I only (B) II only (C) III only
(D) II and III only (E) I, II, and III

2. B

The slope field for the differential equation $\frac{dy}{dx} = \frac{x^2y + y^2}{4x + 2y}$ will have vertical segments when

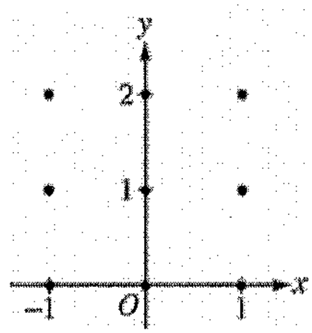
(A) $y = 2x$ (B) $y = -2x$ (C) $y = -x^2$ only (D) $y = 0$ only (E) $y = 0$ or $y = -x^2$

FREE REPSONSE

Your score:_____ out of 7 points

Question 5

Consider the differential equation $\frac{dy}{dx} = \frac{1}{2}x + y - 1$.



(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.
(Note: Use the axes provided in the exam booklet.)

(b) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Describe the region in the xy-plane in which all solution curves to the differential equation are concave up.

(c) Let $y = f(x)$ be a particular solution to the differential equation with the initial condition $f(0) = 1$. Does f have a relative minimum, a relative maximum, or neither at $x = 0$? Justify your answer.

See next page!

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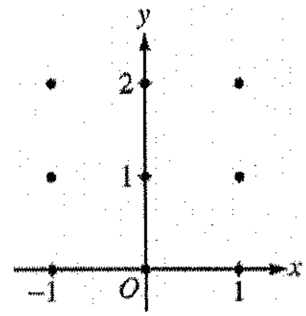
2007 SCORING GUIDELINES (Form B)

Question 5

Consider the differential equation $\frac{dy}{dx} = \frac{1}{2}x + y - 1.$



(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.
(Note: Use the axes provided in the exam booklet.)

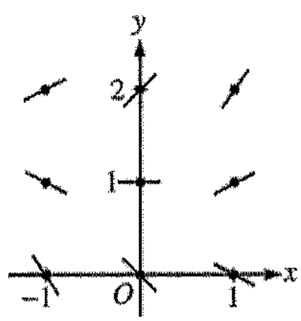
(b) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Describe the region in the xy-plane in which all solution curves to the differential equation are concave up.

(c) Let $y = f(x)$ be a particular solution to the differential equation with the initial condition $f(0) = 1$. Does f have a relative minimum, a relative maximum, or neither at $x = 0$? Justify your answer.

(d) Find the values of the constants m and b, for which $y = mx + b$ is a solution to the differential equation.

Skip

(a)



2 : Sign of slope at each point and relative steepness of slope lines in rows and columns.

(b) $\frac{d^2y}{dx^2} = \frac{1}{2} + \frac{dy}{dx} = \frac{1}{2}x + y - \frac{1}{2}$

Solution curves will be concave up on the half-plane above the line $y = -\frac{1}{2}x + \frac{1}{2}.$

$3 : \begin{cases} 2 : \frac{d^2y}{dx^2} \\ 1 : \text{description} \end{cases}$

(c) $\left.\frac{dy}{dx}\right|_{(0,\,1)} = 0 + 1 - 1 = 0$ and $\left.\frac{d^2y}{dx^2}\right|_{(0,\,1)} = 0 + 1 - \frac{1}{2} > 0$

Thus, f has a relative minimum at $(0, 1)$.

$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

(d) Substituting $y = mx + b$ into the differential equation:

$m = \frac{1}{2}x + (mx + b) - 1 = \left(m + \frac{1}{2}\right)x + (b - 1)$

Then $0 = m + \frac{1}{2}$ and $m = b - 1$: $m = -\frac{1}{2}$ and $b = \frac{1}{2}$.

Skip

$2 : \begin{cases} 1 : \text{value for } m \\ 1 : \text{value for } b \end{cases}$