10.2 u Substitution Indefinite Integrals

PRACTICE

Find the indefinite integral.

1. $\int 5x^3\sqrt{1-x^2}\,dx$

$u = 1-x^2$

$\frac{du}{-2x} = \frac{-2x\,dx}{-2x}$

$\frac{du}{-2x} = dx$

$\int 5x\,u^{1/3}\,dx$

$\int 5x\,u^{1/3}\,\frac{du}{-2x}$

$\int -\frac{5}{2}u^{1/3}\,du$

$-\frac{5}{2}\int u^{1/3}\,du$

$-\frac{5}{2}\cdot\frac{3}{4}u^{4/3}+C$

$-\frac{15}{8}\sqrt[3]{(1-x^2)^4}+C$

2. $\int \frac{x^2}{(1+x^3)^2}\,dx$

$u = 1+x^3$

$\frac{du}{3x^2} = \frac{3x^2\,dx}{3x^2}$

$\frac{du}{3x^2} = dx$

$\int \frac{x^2}{u^2}\,dx$

$\int \frac{x^2}{u^2}\,\frac{du}{3x^2}$

$\frac{1}{3}\int u^{-2}\,du$

$-\frac{1}{3}u^{-1}+C$

$\frac{-1}{3(1+x^3)}+C$

$\frac{-1}{3+3x^3}+C$

3. $\int \sin(2t)\,dt$

$u = 2t$

$\frac{du}{2} = \frac{2\,dt}{2}$

$\frac{du}{2} = dt$

$\int \sin(u)\,dt$

$\int \sin(u)\cdot\frac{du}{2}$

$\frac{1}{2}\int \sin(u)$

$-\frac{1}{2}\cos(u)+C$

$-\frac{1}{2}\cos(2t)+C$

4. $\int \left(x^2-\frac{1}{x^2}\right)dx$

$\int (x^2-x^{-2})\,dx$

$\frac{1}{3}x^3+x^{-1}+C$

$\frac{1}{3}x^3+\frac{1}{x}+C$

Don't need u sub!!!

5. $\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx$

$u = \sqrt{x}$

$du = \frac{1}{2}x^{-\frac{1}{2}}\,dx$

$du = \frac{1}{2\sqrt{x}}\,dx$

$2\sqrt{x}\,du = dx$

$\int \frac{\cos(u)}{\sqrt{x}}\,dx$

$\int \frac{\cos(u)}{\sqrt{x}}\cdot 2\sqrt{x}\,du$

$2\int \cos(u)\,du$

$2\sin(u)+c$

$2\sin(\sqrt{x})+C$

6. $\int xe^{x^2}\,dx$

$u = x^2$

$\frac{du}{2x} = \frac{2x\,dx}{2x}$

$\frac{du}{2x} = dx$

$\int x\,e^u\,dx$

$\int x\,e^u\cdot\frac{du}{2x}$

$\frac{1}{2}\int e^u\,du$

$\frac{1}{2}e^u+C$

$\frac{1}{2}e^{x^2}+C$

7. $\int \frac{\sin x}{1+\cos^2 x}\,dx$

$u = \cos x$

$\frac{du}{-\sin x} = \frac{-\sin x\,dx}{-\sin x}$

$\frac{du}{-\sin x} = dx$

$\int \frac{\sin x}{1+u^2}\,dx$

$\int \frac{\sin x}{1+u^2}\cdot\frac{du}{-\sin x}$

$-\int \frac{1}{1+u^2}\,du$

Snap dawg, that is an inverse trig derivative!

$-\tan^{-1}(u)+C$

$-\tan^{-1}(\cos x)+C$

8. $\int \frac{x^2+x}{x}\,dx$

Rewrite!

$\int \left(\frac{x^2}{x}+\frac{x}{x}\right)dx$

$\int (x+1)\,dx$

$\frac{1}{2}x^2+x+C$

9. $\int 3\sec^2(3\theta)\,d\theta$

$u = 3\theta$

$\frac{du}{3} = \frac{3\,d\theta}{3}$

$\frac{du}{3} = d\theta$

$\int 3\sec^2(u)\,d\theta$

$\int 3\sec^2(u)\,\frac{du}{3}$

$\int \sec^2(u)\,du$

$\tan(u)+C$

$\tan(3\theta)+C$

10. $\int e^x \sin e^x dx$ $\int e^x \sin(u)\, dx$

$u = e^x$

$\frac{du}{e^x} = \frac{e^x dx}{e^x}$ $\int e^x \sin(u) \cdot \frac{du}{e^x}$

$\frac{du}{e^x} = dx$ $\int \sin(u)\, du$

$-\cos(u) + C$

$-\cos(e^x) + C$

11. $\int \tan x \cos x\, dx$ Rewrite!

$\int \frac{\sin x}{\cos x} \cdot \cos x\, dx$

$\int \sin x$

$-\cos x + C$

12. $\int \frac{\sec^2 x}{\sqrt{\tan x}} dx$ $\int \frac{\sec^2 x}{\sqrt{u}} \cdot \frac{du}{\sec^2 x}$

$u = \tan x$

$\frac{du}{\sec^2 x} = \frac{\sec^2 x\, dx}{\sec^2 x}$

$\frac{du}{\sec^2 x} = dx$

$\int u^{-1/2}\, du$

$2u^{1/2} + C$

$2\sqrt{\tan(x)} + C$

13. $\int \sqrt[3]{x}(x^2+1)dx$ Rewrite!

$\int x^{1/3}(x^2+1)\, dx$

$\int (x^{2/3} + x^{1/3})\, dx$

$\frac{3}{5}x^{5/3} + \frac{3}{4}x^{4/3} + C$

14. $\int \frac{x\,dx}{\sqrt{1-x^2}}$ $\int \frac{x}{\sqrt{u}} \cdot dx$

$u = 1 - x^2$

$\frac{du}{-2x} = \frac{-2x\,dx}{-2x}$

$\frac{du}{-2x} = dx$

$\int \frac{x}{u^{1/2}} \cdot \frac{du}{-2x}$

$-\frac{1}{2}\int u^{-1/2}\, du$

$-\frac{1}{2} \cdot 2u^{1/2} + C$

$-\sqrt{1-x^2} + C$

15. $\int r(r^2+1)^{\frac{3}{2}} dr$ $\int r(u)^{3/2} dr$

$u = r^2 + 1$

$\frac{du}{2r} = \frac{2r\,dr}{2r}$

$\frac{du}{2r} = dr$

$\int r(u)^{3/2} \frac{du}{2r}$

$\frac{1}{2}\int u^{3/2}\, du$

$\frac{1}{2} \cdot \frac{2}{5}u^{5/2} + C$

$\frac{1}{5}\sqrt{(r^2+1)^5} + C$

16. $\int \frac{(\ln x)^5}{x} dx$ $\int \frac{u^5}{x} dx$

$u = \ln x$

$du = \frac{1}{x} dx$

$x\,du = dx$

$\int \frac{u^5}{x} x\, du$

$\int u^5\, du$

$\frac{1}{6}u^6 + C$

$\frac{1}{6}(\ln(x))^6 + C$

17. $\int (2x+5)(x^2+5x)^7 dx$

$u = x^2 + 5x$

$\frac{du}{2x+5} = \frac{(2x+5)dx}{2x+5}$

$\frac{du}{2x+5} = dx$

$\int (2x+5)\, u^7 dx$

$\int (2x+5)\, u^7 \cdot \frac{du}{2x+5}$

$\int u^7\, du$

$\frac{1}{8}u^8 + C$

$\frac{1}{8}(x^2+5x)^8 + C$

18. $\int \frac{e^x}{4-e^x} dx$ $\int \frac{e^x}{u} \cdot dx$

$u = 4 - e^x$

$\frac{du}{-e^x} = \frac{-e^x dx}{-e^x}$

$\frac{du}{-e^x} = dx$

$\int \frac{e^x}{u} \cdot \frac{du}{-e^x}$

$-\int \frac{1}{u}\, du$

$-\ln|u| + C$

$-\ln|4-e^x| + C$

MULTIPLE CHOICE

25. $\int x \sin x^2 \, dx =$

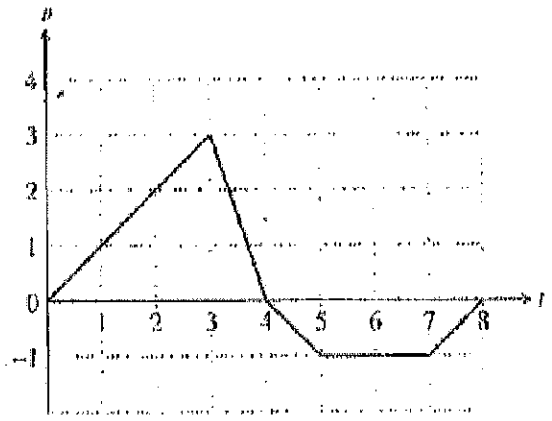
(A) $-\frac{1}{2}\cos x^2 + C$ (B) $\frac{1}{2}\cos x^2 + C$

(C) $-x^2 \cos x^2 + C$ (D) $x^2 \cos x^2 + C$

(E) $\frac{1}{2}x^2 \cos \frac{x^2}{3} + C$

A

Questions 8–9 refer to the following situation.



A spider begins to crawl up a vertical blade of grass at time $t = 0$. The velocity v of the spider at time t, $0 \le t \le 8$, is given by the function whose graph is shown.

8. At what value of t does the spider change direction?

(A) 3 (B) 4 (C) 5

(D) 7 (E) 8

B

9. What is the total distance the spider traveled from $t = 0$ to $t = 8$?

(A) 3 (B) 8 (C) 9

(D) 10 (E) 15

C

6. $\frac{1}{3}\int e^{t/3}\,dt =$

(A) $e^t + C$ (B) $3e^{t/3} + C$ (C) $e^{t/3} + C$

(D) $\frac{1}{3}e^{t/3} + C$ (E) $e^{-2/3t} + C$

C

17. The acceleration of a particle moving along the x-axis at time t is given by $a(t) = 4t - 12$. If the velocity is 10 when $t = 0$ and the position is 4 when $t = 0$, then the particle is changing direction at

(A) $t = 1$
(B) $t = 3$
(C) $t = 5$
(D) $t = 1$ and $t = 5$
(E) $t = 1$ and $t = 3$ and $t = 5$

D

41. $\int \sin^5(2x)\cos(2x)\,dx =$

(A) $\frac{\sin^6 2x}{12} + C$

(B) $\frac{\sin^6 2x}{6} + C$

(C) $\frac{\sin^6 2x}{3} + C$

(D) $\frac{\cos^5 2x}{3} + C$

(E) $\frac{\cos^5 2x}{6} + C$

A

31. $\lim_{h\to 0} \frac{\tan\left(\frac{\pi}{6}+h\right) - \tan\left(\frac{\pi}{6}\right)}{h} =$

B

(A) $\frac{\sqrt{3}}{3}$ (B) $\frac{4}{3}$ (C) $\sqrt{3}$ (D) 0 (E) $\frac{3}{4}$