Evaluate the definite integral.

1. $\int_1^2 \sqrt{5x-1}\,dx = \int_4^9 u^{1/2}\,dx = \frac{1}{5}\int_4^9 u^{1/2}\,du$

$u = 5x-1$

$du = 5\,dx$

$\frac{1}{5}du = dx$

$\frac{1}{5}\left[\frac{2}{3}u^{3/2}\right]_4^9$

$\frac{1}{5}\left[\frac{2}{3}\sqrt{9^3} - \frac{2}{3}\sqrt{4^3}\right]$

$\frac{1}{5}\left[\frac{2}{3}(27) - \frac{2}{3}(8)\right]$

$\frac{1}{5}\left(18 - \frac{16}{3}\right)$

$\frac{1}{5}\left(\frac{54}{3} - \frac{16}{3}\right)$

$\frac{1}{5}\left(\frac{38}{3}\right)$

$\frac{38}{15}$

2. $\int_0^{\frac{\pi}{2}} \sin(2x)\,dx = \int_0^{\pi} \sin(u)\,dx = \frac{1}{2}\int_0^{\pi} \sin(u)\,du$

$u = 2x$

$du = 2dx$

$\frac{1}{2}du = dx$

$\frac{1}{2}\left[-\cos(u)\right]_0^{\pi}$

$\frac{1}{2}\left[-\cos(\pi) - -\cos(0)\right]$

$\frac{1}{2}\left[-(-1) + (1)\right]$

$\frac{1}{2}\left[1+1\right]$

$\frac{1}{2}(2)$

1

3. $\int_0^1 e^x(4-e^x)dx = \int_3^{4-e} e^x \cdot u \cdot dx = -\int_3^{4-e} u\,du = \int_{4-e}^{3} u\,du$

You must flip the integral boundaries because 4 - *e* is less than 3. Don't forget that when you flip the boundaries it makes the integral negative!

$u = 4 - e^x$

$du = -e^x dx$

$-du = e^x dx$

$\frac{1}{2}u^2\Big]_{4-e}^{3}$

$\frac{1}{2}(3)^2 - \frac{1}{2}(4-e)^2$

$\frac{9}{2} - \frac{(4-e)^2}{2}$

$\frac{9-(4-e)^2}{2}$

4. $\int_0^1 \frac{x}{(x^2+1)^3}\,dx = \int_1^2 \frac{x}{u^3}\,dx = \frac{1}{2}\int_1^2 \frac{du}{u^3} = \frac{1}{2}\int_1^2 u^{-3}\,du$

$u = x^2+1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$\frac{1}{2}\left[-\frac{1}{2}u^{-2}\right]_1^2$

$\frac{1}{2}\left[-\frac{1}{2(2)^2} - -\frac{1}{2(1)^2}\right]$

$\frac{1}{2}\left[-\frac{1}{8} + \frac{1}{2}\right]$

$\frac{1}{2}\left[-\frac{1}{8} + \frac{4}{8}\right]$

$\frac{1}{2}\left(\frac{3}{8}\right)$

$\frac{3}{16}$

5. $\int_{-2}^{3} \frac{1}{1+9t^2}\,dt = \frac{\tan^{-1}(3t)}{3}\Big]_{-2}^{3} = \frac{1}{3}\left(\tan^{-1}(9) - \tan^{-1}(-6)\right)$

CAREFUL this is NOT *u* substitution for 1 + 9t^2

That's right, Mr. Brust put in an inverse trig function just to keep it real!

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6. $\int_0^1 x\sqrt{1-x^2}\,dx = \int_1^0 x\cdot u^{1/2}\cdot dx = -\frac{1}{2}\int_1^0 u^{1/2}\,dx = \frac{1}{2}\int_0^1 u^{1/2}\,dx$

You must flip the integral boundaries because 0 is less than 1. Don't forget that when you flip the boundaries it makes the integral negative!

$u = 1 - x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$\frac{1}{2}\left[\frac{2}{3}u^{3/2} - \frac{2}{3}u^{3/2}\right]_0^1$

$\frac{1}{2}\left[\frac{2}{3}\sqrt{1^3} - \frac{2}{3}\sqrt{0^3}\right]$

$\frac{1}{2}\left(\frac{2}{3}\right) = \frac{1}{3}$

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7. $\int_3^6 \left(\frac{x^2-2x}{x}\right)dx = \int_3^6 \left(\frac{x^2}{x} - \frac{2x}{x}\right)dx = \int_3^6 (x-2)\,dx = \frac{1}{2}x^2 - 2x\Big]_3^6$

rewrite!

$\left(\frac{1}{2}(6)^2 - 2(6)\right) - \left(\frac{1}{2}(3)^2 - 2(3)\right)$

$(18-12) - \left(\frac{9}{2} - 6\right)$

$6 - \left(-\frac{3}{2}\right)$

$6 + \frac{3}{2} = \frac{15}{2}$

CAREFUL this is NOT *u* substitution for x^2-2*x*

Oh know, Mr. Brust did it again. Keep it real yo!

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8. $\int_{-\frac{\pi}{4}}^{0} \tan x \sec^2 x\,dx = \int_{-1}^{0} u\cdot\sec^2 x\cdot dx = \int_{-1}^{0} u\cdot du$

$u = \tan x$

$du = \sec^2 x\,dx$

$\frac{1}{2}u^2\Big]_{-1}^{0}$

$\frac{1}{2}(0)^2 - \frac{1}{2}(-1)^2$

$0 - \frac{1}{2} = -\frac{1}{2}$

9. $\int_0^{\frac{\pi}{8}} \sec(2x)\tan(2x)\,dx = \int_0^{\pi/4} \sec(u)\tan(u)\,dx = \frac{1}{2}\int_0^{\pi/4} \sec(u)\tan(u)\,du$

$u = 2x$

$du = 2\,dx$

$\frac{1}{2}du = dx$

$\frac{1}{2}\left[\sec(u)\right]_0^{\pi/4}$

$\frac{1}{2}\left[\sec\left(\frac{\pi}{4}\right) - \sec(0)\right]$

$\frac{1}{2}\left[\frac{1}{\cos(\pi/4)} - \frac{1}{\cos(0)}\right]$

$\frac{1}{2}\left[\frac{1}{\frac{\sqrt{2}}{2}} - \frac{1}{1}\right] = \frac{1}{2}\left[\frac{2}{\sqrt{2}} - 1\right] = \frac{1}{\sqrt{2}} - \frac{1}{2} = \frac{\sqrt{2}}{2} - \frac{1}{2}$

$= \frac{\sqrt{2}-1}{2}$

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10. $\int_1^e \frac{\ln x}{x}\,dx = \int_{\ln 1}^{\ln e} \frac{u}{x}\,dx = \int_0^1 u\,du$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$\frac{1}{2}u^2\Big]_0^1$

$\frac{1}{2}(1)^2 - \frac{1}{2}(0)^2$

$\frac{1}{2} - 0$

$\frac{1}{2}$

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11. $\int_3^{\sqrt{30}} \frac{2x}{\sqrt{x^2-5}}\,dx = \int_4^{25} \frac{2x}{\sqrt{u}}\,dx = \int_4^{25} u^{-1/2}\,du$

$u = x^2 - 5$

$du = 2x\,dx$

$2u^{1/2}\Big]_4^{25}$

$2\sqrt{25} - 2\sqrt{4}$

$2(5) - 2(2)$

$10 - 4$

6

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12. $\int_0^1 \frac{x^2+2x}{\sqrt[3]{x^3+3x^2+4}}\,dx = \int_4^8 \frac{x^2+2x}{\sqrt[3]{u}}\,dx = \frac{1}{3}\int_4^8 u^{-1/3}\,du$

$u = x^3 + 3x^2 + 4$

$du = (3x^2 + 6x)\,dx$

$\frac{1}{3}du = (x^2 + 2x)\,dx$

$\frac{1}{3}\left[\frac{3}{2}u^{2/3}\right]_4^8$

$\frac{1}{3}\left[\frac{3}{2}\sqrt[3]{8^2} - \frac{3}{2}\sqrt[3]{4^2}\right]$

$\frac{1}{3}\left[\frac{3}{2}(4) - \frac{3}{2}\sqrt[3]{16}\right]$

$\frac{1}{3}\left[6 - \frac{3}{2}\sqrt[3]{16}\right]$

$\frac{1}{3}\left(6 - \frac{3\sqrt[3]{16}}{2}\right)$

$2 - \frac{\sqrt[3]{16}}{2}$

13. $\displaystyle\int_0^{\pi}(2\sin x+\sin 2x)dx = \int_0^{\pi} 2\sin x\,dx + \int_0^{\pi}\sin(2x)\,dx$

$$\left[-2\cos x\right]_0^{\pi} + \left[-\tfrac{1}{2}\cos(2x)\right]_0^{\pi}$$

$$\left(-2\cos(\pi) - -2\cos(0)\right) + \left(-\tfrac{1}{2}\cos(2\pi) - -\tfrac{1}{2}\cos(0)\right)$$

$$\left(-2(-1)+2(1)\right) + \left(-\tfrac{1}{2}(1)+\tfrac{1}{2}(1)\right)$$

$$4 \quad + \quad 0$$

$$4$$

Seriously Mr. Brust?
Tots 4 sers.

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14. $\displaystyle\int_0^{4}\frac{x}{\sqrt{2x+1}}dx = \int_1^9 \frac{x}{\sqrt{u}}\,dx = \frac{1}{2}\int_1^9\left(\frac{u-1}{2}\right)u^{-1/2}du = \frac{1}{4}\int_1^9 (u-1)u^{-1/2}du = \frac{1}{4}\int_1^9 u^{1/2}-u^{-1/2}du$

$$\boxed{u = 2x+1}$$
$$-1 \qquad -1$$
$$\frac{u-1}{2} = \frac{2x}{2}$$
$$\boxed{\frac{u-1}{2} = x}$$

$du = 2\,dx$

$\frac{1}{2}du = dx$

$$\frac{1}{4}\left[\frac{2}{3}u^{3/2} - 2u^{1/2}\right]_1^9$$

$$\frac{1}{4}\left[\left(\frac{2}{3}\sqrt{9^3}-2\sqrt{9}\right)-\left(\frac{2}{3}\sqrt{1^3}-2\sqrt{1}\right)\right]$$

$$\frac{1}{4}\left[(18-6)-\left(\frac{2}{3}-2\right)\right]$$

$$\frac{1}{4}\left[12-\left(-\frac{4}{3}\right)\right] = \frac{1}{4}\left(\frac{40}{3}\right) = \frac{10}{3}$$

If you got this right, you are a boss.
Don't sweat it if you missed it.

MULTIPLE CHOICE

TEST PREP

1. **C**
2. **B**

FREE RESPONSE

(a) $f'(x) = \frac{1}{2}\left(25-x^2\right)^{-1/2}(-2x) = \dfrac{-x}{\sqrt{25-x^2}}, \quad -5 < x < 5$

$2:\ f'(x)$

(b) $f'(-3) = \dfrac{3}{\sqrt{25-9}} = \dfrac{3}{4}$

$f(-3) = \sqrt{25-9} = 4$

An equation for the tangent line is $y = 4 + \frac{3}{4}(x+3)$.

$2: \begin{cases} 1:\ f'(-3) \\ 1:\ \text{answer} \end{cases}$

(c) $\lim\limits_{x\to -3^-} g(x) = \lim\limits_{x\to -3^-} f(x) = \lim\limits_{x\to -3^-}\sqrt{25-x^2} = 4$

$\lim\limits_{x\to -3^+} g(x) = \lim\limits_{x\to -3^+}(x+7) = 4$

Therefore, $\lim\limits_{x\to -3} g(x) = 4$.

$g(-3) = f(-3) = 4$

So, $\lim\limits_{x\to -3} g(x) = g(-3)$.

Therefore, g is continuous at $x = -3$.

$2: \begin{cases} 1:\ \text{considers one-sided limits} \\ 1:\ \text{answer with explanation} \end{cases}$

(d) Let $u = 25 - x^2 \ \Rightarrow\ du = -2x\,dx$

$\displaystyle\int_0^5 x\sqrt{25-x^2}\,dx = -\frac{1}{2}\int_{25}^{0}\sqrt{u}\,du = \left[-\frac{1}{2}\cdot\frac{2}{3}u^{3/2}\right]_{u=25}^{u=0}$

$\displaystyle = -\frac{1}{3}(0-125) = \frac{125}{3}$

$3: \begin{cases} 2:\ \text{antiderivative} \\ 1:\ \text{answer} \end{cases}$