10.4 Separation of Variables

NOTES

CALCULUS

Write your questions here!

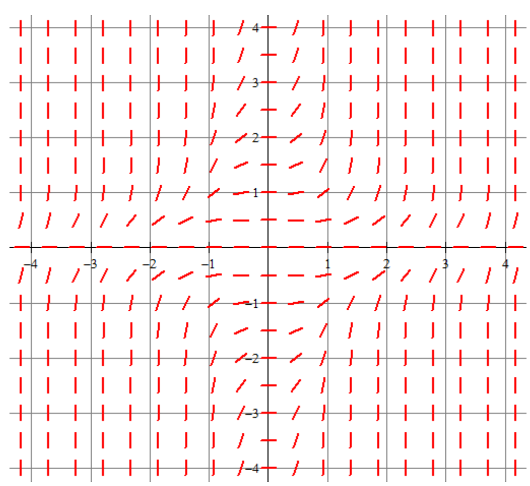
Solve the differential equation.

$$\frac{dy}{dx} = \frac{x^2}{y}$$

$$\frac{dy}{dx} = (\sin x)y^2$$

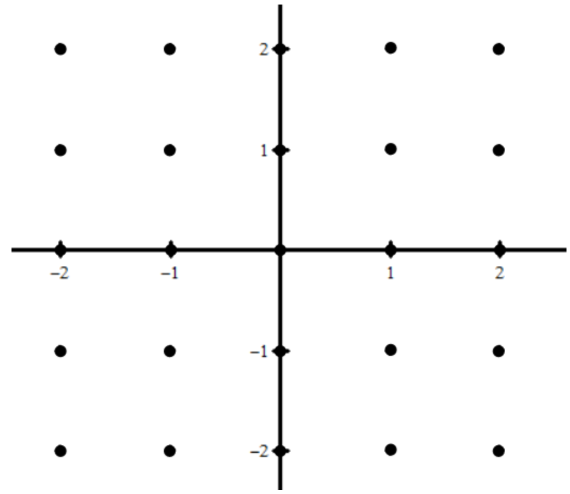
Initial Value

Solve for y if $\frac{dy}{dx} = (xy)^2$ and $y = 1$ when $x = 1$



Use the differential to answer the following:

$$\frac{dy}{dx} = \frac{2x}{y}$$



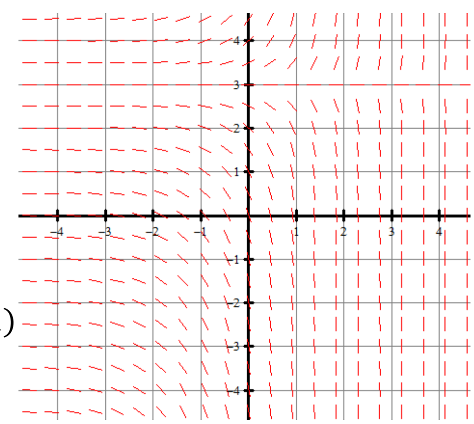
(a) Fill in the slope field

(b) Write the equation of the line tangent to the solution curve at point $(2,1)$.

(c) Find the particular solution with initial condition of $f(2) = 1$.

Solve the differential equation.

$$\frac{dy}{dx} = (y+2)e^{x}$$



(a) Sketch a particular solution through the point $(0, -1)$

(b) Find the particular solution with initial condition $(0, -1)$

SUMMARY:

Now, summarize your notes here!

10.4 Separation of Variables

PRACTICE

Solve the differential equation.

1. $\frac{dy}{dx} = \frac{3x^2}{y}$

2. $\frac{dy}{dx} = e^x y^2$

3. $\frac{dy}{dx} = -2x(y-3)$

4. $\frac{dy}{dx} = x\cos x^2$

Find the solution that satisfies the given condition.

5. $\frac{dy}{dx} = y\sin x$ if $y(0) = 2$

6. $\frac{dy}{dx} = \frac{e^x}{y}$ if $y(0) = -4$

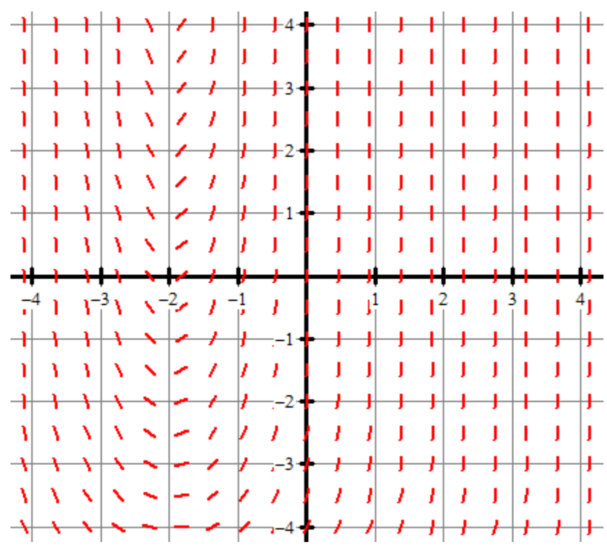
Find the solution that satisfies the given condition.

7. $\frac{dy}{dx} = xy^2$ and $y = 1$ when $x = 0$

8. $\frac{dy}{dx} = \frac{1}{5}(8 - y)$ and $y = 6$ when $x = 0$

Use the differential equation and its slope field to answer the following.

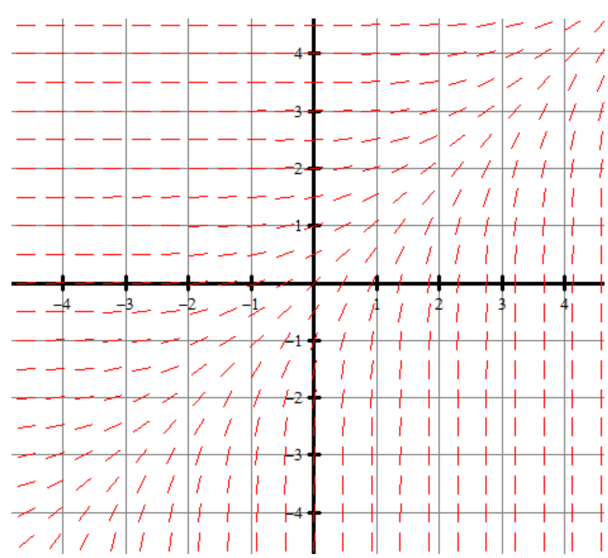
9. $\frac{dy}{dx} = (y + 5)(x + 2)$



   a. Sketch a particular solution through the point $(0,1)$.

   b. Find the particular solution $y = f(x)$ when $f(0) = 1$

10. $\frac{dy}{dx} = e^{x-y}$



   a. Sketch a particular solution through the point $(0,2)$.

   b. Find the particular solution $y = f(x)$ when $f(0) = 2$

MULTIPLE CHOICE

1. $\int_{-1}^{1} \frac{4}{1+x^2}\, dx =$

(A) 0
(B) π
(C) 1
(D) 2π
(E) 2

2. If $\frac{dy}{dx} = \frac{(3x^2+2)}{y}$ and $y = 4$ when $x = 2$, then when $x = 3$, $y =$

(A) 18
(B) $\pm\sqrt{66}$
(C) 58
(D) $\pm\sqrt{74}$
(E) $\pm\sqrt{58}$

3. If $\frac{dy}{dx} = \frac{x^3+1}{y}$ and $y = 2$ when $x = 1$, then when $x = 2$, $y =$

(A) $\sqrt{\frac{27}{2}}$
(B) $\sqrt{\frac{27}{8}}$
(C) $\pm\sqrt{\frac{27}{8}}$
(D) $\pm\frac{3}{2}$
(E) $\pm\sqrt{\frac{27}{2}}$

4. If $\frac{dy}{dt} = -2y$ and if $y = 1$ when $t = 0$, what is the value of t for which $y = \frac{1}{2}$?

(A) $-\frac{1}{2}\ln 2$
(B) $-\frac{1}{4}$
(C) $\frac{1}{2}\ln 2$
(D) $\frac{\sqrt{2}}{2}$
(E) $\ln 2$

5. What is the equation of the line tangent to the graph $y = \sin^2 x$ at $x = \frac{\pi}{4}$?

(A) $y - \frac{1}{2} = -\left(x - \frac{\pi}{4}\right)$

(B) $y - \frac{1}{2} = \left(x - \frac{\pi}{4}\right)$

(C) $y - \frac{1}{\sqrt{2}} = \left(x - \frac{\pi}{4}\right)$

(D) $y - \frac{1}{\sqrt{2}} = \frac{1}{2}\left(x - \frac{\pi}{4}\right)$

(E) $y - \frac{1}{2} = \frac{1}{2}\left(x - \frac{\pi}{4}\right)$

FREE RESPONSE **YOUR SCORE:____ out of 9**

6. Consider the differential equation $\frac{dy}{dx} = e^y(3x^2 - 6x)$. Let $y = f(x)$ be the particular solution to the differential equation that passes throug$(1,0)$h.

(a) Write an equation for the line tangent to the graph of *f* at the point $(1,0)$. Use the tangent line to approximate $f(1.2)$.

(b) Find $y = f(x)$, the particular solution to the differential equation that passes through $(1,0)$.