PRACTICE

Solve the differential equation.

1. $\frac{dy}{dx} = \frac{3x^2}{y}$

$y\,dy = 3x^2\,dx$

$\int y\,dy = \int 3x^2\,dx$

$\frac{1}{2}y^2 = x^3 + C$

$y^2 = 2x^3 + C$

$y = \pm\sqrt{2x^3 + C}$

2. $\frac{dy}{dx} = e^x y^2$

$\frac{1}{y^2}\,dy = e^x\,dx$

$\int y^{-2}\,dy = \int e^x\,dx$

$-y^{-1} = e^x + C$

$-\frac{1}{y} = e^x + C$

$\frac{1}{y} = -e^x + C$

$y = \frac{1}{-e^x + C}$

3. $\frac{dy}{dx} = -2x(y-3)$

$\frac{1}{y-3}\,dy = -2x\,dx$

$\int \frac{1}{y-3}\,dy = \int -2x\,dx$

$\ln|y-3| = -x^2 + C$

$e^{\ln|y-3|} = e^{-x^2+C}$

$y - 3 = e^{-x^2+C}$

$y - 3 = e^{-x^2}e^{C}$

$y - 3 = Ce^{-x^2}$

$y = Ce^{-x^2} + 3$

4. $\frac{dy}{dx} = x\cos x^2$

$dy = x\cos x^2\,dx$

$\int dy = \int x\cos x^2\,dx$

$u = x^2$

$du = 2x\,dx$

$y = \frac{1}{2}\int \cos u\,du$

$y = \frac{1}{2}\sin x^2 + C$

Find the solution that satisfies the given condition.

5. $\frac{dy}{dx} = y \sin x$ if $y(0) = 2$

$\frac{1}{y}\,dy = \sin x\,dx$

$\int \frac{1}{y}\,dy = \int \sin x\,dx$

$\ln|y| = -\cos x + C$

$\ln 2 = -\cos(0) + C$

$\ln 2 = -1 + C$

$\ln 2 + 1 = C$

$\ln|y| = -\cos x + C$

$e^{\ln|y|} = e^{-\cos x + C}$

$y = e^{-\cos x + C}$

$y = e^{-\cos x} e^{C}$

$y = C e^{-\cos x}$

$\boxed{y = (\ln 2 + 1)e^{-\cos x}}$

6. $\frac{dy}{dx} = \frac{e^x}{y}$ if $y(0) = -4$

$y\,dy = e^x\,dx$

$\int y\,dy = \int e^x\,dx$

$\frac{1}{2}y^2 = e^x + C$

$\frac{1}{2}(-4)^2 = e^0 + C$

$8 = 1 + C$

$7 = C$

$\frac{1}{2}y^2 = e^x + 7$

$y^2 = 2e^x + 14$

$y = \pm\sqrt{2e^x + 14}$

$y = -\sqrt{2e^x + 14}$

Note: When you have a particular solution you must chose if it is the positive solution or the negative solution. In this case negative because the *y*-value is negative

7. $\frac{dy}{dx} = xy^2$ and $y = 1$ when $x = 0$

$\frac{1}{y^2}\,dy = x\,dx$

$\int \frac{1}{y^2}\,dy = \int x\,dx$

$-y^{-1} = \frac{1}{2}x^2 + C$

$-\frac{1}{1} = \frac{1}{2}(0)^2 + C$

$-1 = 0 + C$

$-1 = C$

$-\frac{1}{y} = \frac{1}{2}x^2 - 1$

$\frac{1}{y} = -\frac{1}{2}x^2 + 1$

$\boxed{y = \frac{1}{-\frac{1}{2}x^2 + 1}}$

8. $\frac{dy}{dx} = \frac{1}{5}(8 - y)$ and $y = 6$ when $x = 0$

$\frac{1}{8-y}\,dy = \frac{1}{5}\,dx$

$\int \frac{1}{8-y}\,dy = \int \frac{1}{5}\,dx$

$-\ln|8-y| = \frac{1}{5}x + C$

$-\ln|8-6| = \frac{1}{5}(0) + C$

$-\ln 2 = C$

$-\ln|8-y| = \frac{1}{5}x - \ln 2$

$\ln|8-y| = -\frac{1}{5}x + \ln 2$

$e^{\ln|8-y|} = e^{-\frac{1}{5}x + \ln 2}$

$8 - y = e^{-1/5\,x} \cdot e^{\ln 2}$

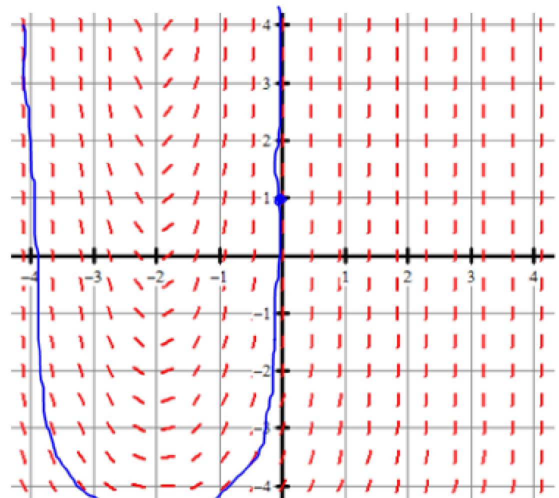
$8 - y = 2e^{-1/5\,x}$

$-y = 2e^{-1/5\,x} - 8$

$y = -2e^{-1/5\,x} + 8$

Use the differential equation and its slope field to answer the following.

9. $\frac{dy}{dx} = (y+5)(x+2)$



$\frac{1}{y+5}\,dy = (x+2)\,dx$

$\int \frac{1}{y+5}\,dy = \int (x+2)\,dx$

$\ln|y+5| = \frac{1}{2}x^2 + 2x + C$

a. Sketch a particular solution through the point (0,1).

b. Find the particular solution $y = f(x)$ when $f(0) = 1$

$\ln|1+5| = \frac{1}{2}(0)^2 + 2(0) + C$

$\ln 6 = C$

$\ln|y+5| = \frac{1}{2}x^2 + 2x + \ln 6$

$e^{\ln|y+5|} = e^{\frac{1}{2}x^2 + 2x + \ln 6}$

$y+5 = e^{\frac{1}{2}x^2} \cdot e^{2x} \cdot e^{\ln 6}$

$y+5 = 6e^{\frac{1}{2}x^2 + 2x}$

$y = 6e^{(\frac{1}{2}x^2 + 2x)} - 5$

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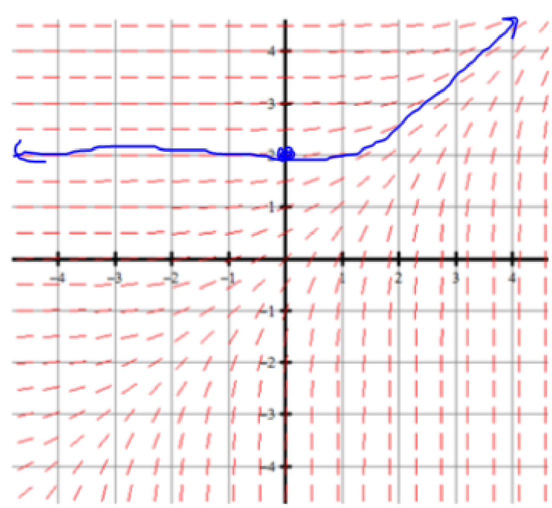
10. $\frac{dy}{dx} = e^{x-y}$ $\quad \frac{dy}{dx} = \frac{e^x}{e^y}$



$e^y\,dy = e^x\,dx$

$\int e^y\,dy = \int e^x\,dx$

$e^y = e^x + C$

a. Sketch a particular solution through the point (0,2).

b. Find the particular solution $y = f(x)$ when $f(0) = 2$

$e^2 = e^0 + C$

$e^2 = 1 + C$

$e^2 - 1 = C$

$e^y = e^x + e^2 - 1$

$\ln e^y = \ln(e^x + e^2 - 1)$

$y = \ln(e^x + e^2 - 1)$

MULTIPLE CHOICE

1. D
2. E
3. E
4. C $\longrightarrow$ Note: I got $-\frac{1}{2}\ln\left(\frac{1}{2}\right)$ which you can write as $\frac{1}{2}\ln\left(\frac{1}{2}\right)^{-1}$ which is $\frac{1}{2}\ln(2)$
5. B

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Question 6

Consider the differential equation $\frac{dy}{dx} = e^y\left(3x^2 - 6x\right)$. Let $y = f(x)$ be the particular solution to the differential equation that passes through $(1, 0)$.

(a) Write an equation for the line tangent to the graph of f at the point $(1, 0)$. Use the tangent line to approximate $f(1.2)$.

(b) Find $y = f(x)$, the particular solution to the differential equation that passes through $(1, 0)$.

(a) $\left.\frac{dy}{dx}\right|_{(x,\,y)=(1,\,0)} = e^0\left(3\cdot 1^2 - 6\cdot 1\right) = -3$

An equation for the tangent line is $y = -3(x-1)$.

$f(1.2) \approx -3(1.2-1) = -0.6$

$$3 : \begin{cases} 1 : \frac{dy}{dx} \text{ at the point } (x, y) = (1, 0) \\ 1 : \text{tangent line equation} \\ 1 : \text{approximation} \end{cases}$$

(b) $\frac{dy}{e^y} = \left(3x^2 - 6x\right) dx$

$$\int \frac{dy}{e^y} = \int \left(3x^2 - 6x\right) dx$$

$$-e^{-y} = x^3 - 3x^2 + C$$

$$-e^{-0} = 1^3 - 3\cdot 1^2 + C \Rightarrow C = 1$$

$$-e^{-y} = x^3 - 3x^2 + 1$$

$$e^{-y} = -x^3 + 3x^2 - 1$$

$$-y = \ln\left(-x^3 + 3x^2 - 1\right)$$

$$y = -\ln\left(-x^3 + 3x^2 - 1\right)$$

Note: This solution is valid on an interval containing $x = 1$ for which $-x^3 + 3x^2 - 1 > 0$.

$$6 : \begin{cases} 1 : \text{separation of variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$$

Note: max $3/6$ [1-2-0-0-0] if no constant of integration

Note: $0/6$ if no separation of variables