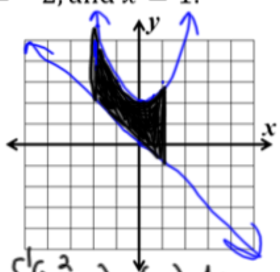
11.1 Area Between Curves

Calculus

Practice

Sketch **the graph of each equation, then set up the integral to find the area of the region bounded by the graphs. Do NOT evaluate, just set up the integral!**

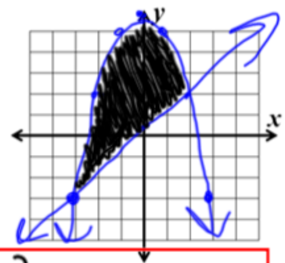
1. $f(x) = x^2 + 2,\ g(x) = -x,$ $x = -2,$ and $x = 1.$



$\int_{-2}^{1} (x^2+2)-(-x)\,dx$

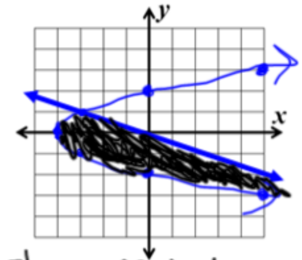
$\boxed{\int_{-2}^{1} x^2+x+2\,dx}$

2. $f(x) = 6 - x^2$ and $g(x) = x$



$\boxed{\int_{-3}^{2} 6-x^2-x\,dx}$

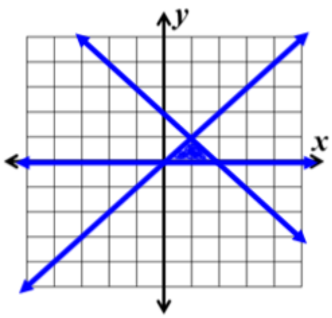
3. $f(y) = y^2 - 4,\ g(y) = -3y$



$\int_{-4}^{1} -3y-(y^2-4)\,dy$

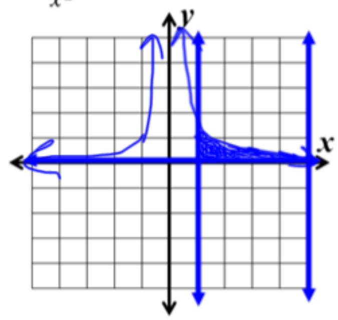
$\boxed{\int_{-4}^{1} -3y-y^2+4\,dy}$

4. $y = x,\ y = 2 - x,\ y = 0$



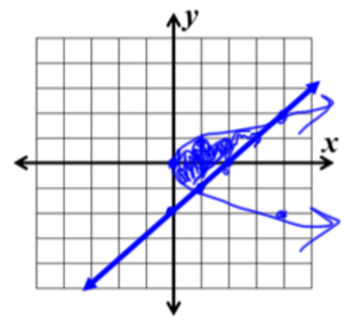
$\boxed{\int_{0}^{1} x\,dx + \int_{1}^{2} 2-x\,dx}$

5. $y = \frac{1}{x^2},\ y = 0,\ x = 1,\ x = 5$



$\boxed{\int_{1}^{5} \frac{1}{x^2}\,dx}$

6. $f(y) = y^2,\ g(y) = y + 2$



$\boxed{\int_{-1}^{2} y+2-y^2\,dy}$

7. $f(x) = 2x^3 - x^2 - 7x,\ g(x) = x^2 + 5x$

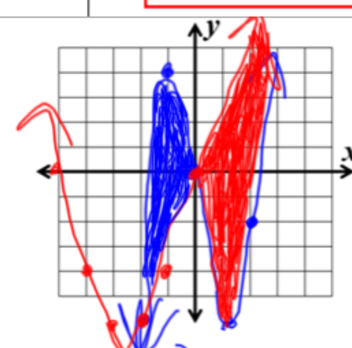
$2x^3 - x^2 - 7x = x^2 + 5x$

$2x^3 - 2x^2 - 12x = 0$

$2x(x^2 - x - 6) = 0$

$2x(x-3)(x+2) = 0$

$x = 0 \quad x = 3 \quad x = -2$



$\int_{-2}^{0} 2x^3-2x^2-12x\,dx + \int_{0}^{3} -2x^3+2x^2+12x\,dx$

Find the area of the region bounded by the given equations. Evaluate an integral with respect to *x* (perpendicular to the *x*-axis) by using a calculator. Find the same area by evaluating an integral with respect to *y* (perpendicular to the *y*-axis) by showing your work.

8. $y = x^2$ and $y = x^3$

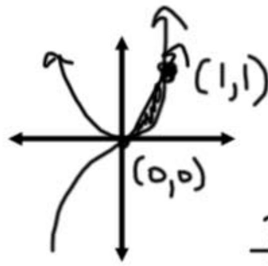
Sketch your graph here in the middle!

with respect to *x* (and a calculator)

$$\int_0^1 x^2 - x^3\,dx = \boxed{\frac{1}{12}}$$

or

0.833



with respect to *y* (show work)

$\sqrt{y} = x$ $\qquad$ $\sqrt[3]{y} = x$

$$\int_0^1 y^{\frac{1}{3}} - y^{\frac{1}{2}}\,dy$$

$$\frac{3y^{\frac{4}{3}}}{4} - \frac{2y^{\frac{3}{2}}}{3}\Big|_0^1$$

$$\frac{3}{4} - \frac{2}{3} - (0) = \frac{9}{12} - \frac{8}{12} = \boxed{\frac{1}{12}}$$

9. $y = \sqrt{x}, x = 0$ and $y = x - 2$

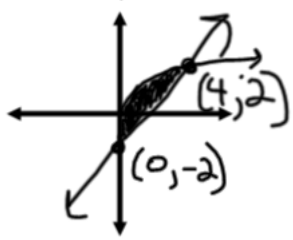
Sketch your graph here in the middle!

with respect to *x* (and a calculator)

$$\int_0^4 \sqrt{x} - (x-2)\,dx = \boxed{\frac{16}{3}}$$

or

5.333



$\sqrt{x} = x - 2$

$x = (x-2)^2$

$0 = x^2 - 5x + 4$

$0 = (x-4)(x-1)$

with respect to *y* (show work)

$y^2 = x$ $\qquad$ $y + 2 = x$

$$\int_{-2}^0 y+2\,dy + \int_0^2 y+2-y^2\,dy$$

$$\frac{y^2}{2} + 2y\Big|_{-2}^0 + \frac{y^2}{2} + 2y - \frac{y^3}{3}\Big|_0^2$$

$$0 - (2-4) + \left(2 + 4 - \frac{8}{3}\right) - (0)$$

$$2 + \frac{10}{3} = \boxed{\frac{16}{3}}$$

10. $y = x^2, y = -x, x = 0, x = 2$

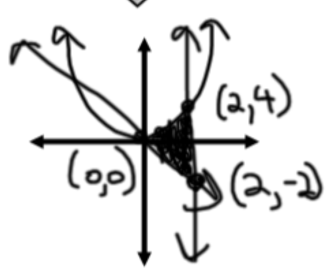
Sketch your graph here in the middle!

with respect to *x* (and a calculator)

$$\int_0^2 x^2 + x\,dx = \boxed{\frac{14}{3}}$$

or

4.6667



with respect to *y* (show work)

$\sqrt{y} = x$ $\qquad$ $-y = x$

$$\int_{-2}^0 2-(-y)\,dy + \int_0^4 2-\sqrt{y}\,dy$$

$$2y + \frac{y^2}{2}\Big|_{-2}^0 + 2y - \frac{2y^{\frac{3}{2}}}{3}\Big|_0^4$$

$$0 - (-4+2) + \left(8 - \frac{16}{3}\right) - (0)$$

$$2 + 8 - \frac{16}{3}$$

$$\frac{30}{3} - \frac{16}{3} = \boxed{\frac{14}{3}}$$

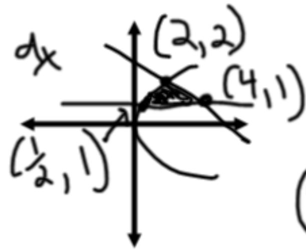
11. $x = \frac{1}{2}y^2, y = 1, y = -\frac{1}{2}x + 3$ *Sketch* your graph here in the middle!

with respect to x (and a calculator)

$y = \pm\sqrt{2x}$

$\int_{1/2}^{2} \sqrt{2x} - 1\, dx + \int_{2}^{4} \left(-\frac{1}{2}x+3\right) - 1\, dx$

$\boxed{\frac{11}{6}}$ or 1.833



with respect to y (show work)

$6 - 2y = x$

$\int_{1}^{2} 6 - 2y - \frac{1}{2}y^2\, dy$

$6y - y^2 - \frac{y^3}{6} \Big|_1^2$

$\left(12 - 4 - \frac{8}{6}\right) - \left(6 - 1 - \frac{1}{6}\right)$

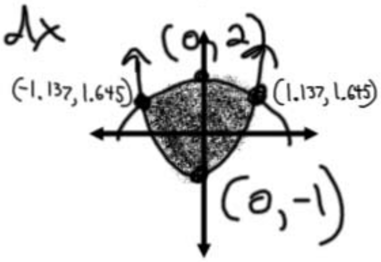
$\frac{20}{3} - \frac{29}{6} = \boxed{\frac{11}{6}}$

12. $y = e^{x^2} - 2$ and $y = \sqrt{4 - x^2}$ *Sketch* your graph here in the middle!

with respect to x (and a calculator)

$\int_{-1.137}^{1.137} \sqrt{4 - x^2} - \left(e^{x^2} - 2\right) dx$

$\boxed{5.0496}$



with respect to y (~~show work~~) Calculator!

$x = \pm\sqrt{\ln(y+2)}$ $\quad x = \pm\sqrt{4 - y^2}$

$\int_{-1}^{1.645} \sqrt{\ln(y+2)} - -\sqrt{\ln(y+2)}\, dy + \int_{1.645}^{2} \sqrt{4-y^2} - -\sqrt{4-y^2}\, dy$

$\boxed{5.061}$

Why are these answers different? The boundaries were rounded, creating a rounding error for your final answer.

Test Prep: 1C, 2C, 3D

2003 Form A #1

Point of intersection $e^{-3x} = \sqrt{x}$ at $(T, S) = (0.238734, 0.488604)$ (a) Area $= \int_T^1 \left(\sqrt{x} - e^{-3x}\right) dx$ $= 0.442$ or 0.443	1: Correct limits in an integral in (a), (b), or (c) 2 : { 1 : integrand 1 : answer

2003 Form B #1

(a) $f'(x) = 8x - 3x^2$; $f'(3) = 24 - 27 = -3$

$f(3) = 36 - 27 = 9$

Tangent line at $x = 3$ is

$y = -3(x - 3) + 9 = -3x + 18,$

which is the equation of line ℓ.

$$2 : \begin{cases} 1 : \text{finds } f'(3) \text{ and } f(3) \\ 1 : \begin{cases} \text{finds equation of tangent line} \\ \qquad\text{or} \\ \text{shows (3,9) is on both the} \\ \text{graph of } f \text{ and line } \ell \end{cases} \end{cases}$$

(b) $f(x) = 0$ at $x = 4$

The line intersects the x-axis at $x = 6$.

$$\text{Area } = \frac{1}{2}(3)(9) - \int_3^4 \left(4x^2 - x^3\right) dx$$
$$= 7.916 \text{ or } 7.917$$

OR

$$\text{Area } = \int_3^4 \left((18 - 3x) - \left(4x^2 - x^3\right)\right) dx + \frac{1}{2}(2)(18 - 12)$$
$$= 7.916 \text{ or } 7.917$$

$$4 : \begin{cases} 2 : \text{integral for non-triangular region} \\ \quad 1 : \text{limits} \\ \quad 1 : \text{integrand} \\ 1 : \text{area of triangular region} \\ 1 : \text{answer} \end{cases}$$