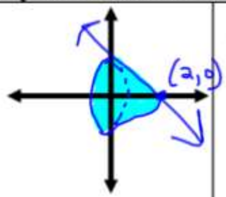
11.2 Solids of Revolution (Disks)

Calculus

Practice

For each problem, sketch the area bounded by the equations and revolve it around the x-axis. Find the volume of the resulting solid formed by this revolution. Leave your answers in terms of π.

1. $y = -x + 2,\ x = 0,\ y = 0$



$\pi\int_0^2 (-x+2)^2\,dx$

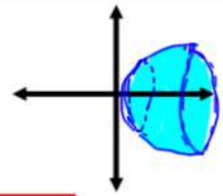
$\pi\int_0^2 (x^2-4x+4)\,dx$

$\pi\left[\frac{x^3}{3}-2x^2+4x\right]\Big|_0^2$

$\pi\left[\left(\frac{8}{3}-8+8\right)-(0)\right]$

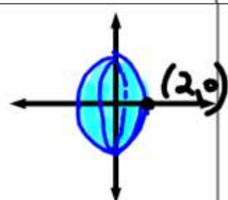
$\boxed{\frac{8}{3}\pi}$

2. $y = \sqrt{x},\ x = 1,\ x = 4$



$\pi\int_1^4 (\sqrt{x})^2\,dx = \boxed{\frac{15}{2}\pi}$

3. $y = 4 - x^2,\ y = 0,\ x \geq 0$



$\pi\int_0^2 (4-x^2)^2\,dx$

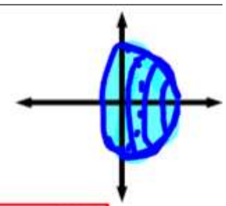
$\pi\int_0^2 (16-8x^2+x^4)\,dx$

$\pi\left[16x-\frac{8}{3}x^3+\frac{x^5}{5}\right]\Big|_0^2$

$\pi\left[\left(32-\frac{64}{3}+\frac{32}{5}\right)-(0)\right]$

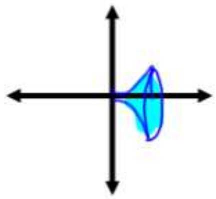
$\pi\left(\frac{480}{15}-\frac{320}{15}+\frac{96}{15}\right) = \boxed{\frac{256}{15}\pi}$

4. $y = \sqrt{9 - x^2},\ x = 0,\ y = 0$



$\pi\int_0^3 (\sqrt{9-x^2})^2\,dx = \boxed{18\pi}$

5. $y = x^3,\ y = 0,\ x = 2$

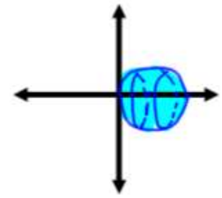


$\pi\int_0^2 (x^3)^2 dx$

$\pi\int_0^2 x^6\, dx$

$\pi \frac{x^7}{7}\Big|_0^2 = \boxed{\frac{128}{7}\pi}$

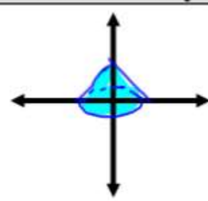
6. $y = \sqrt{\sin x}, y = 0, x = 0, x = \pi$



$\pi\int_0^\pi (\sqrt{\sin x})^2 dx = \boxed{2\pi}$

Same instructions as above but revolve around the *y*-axis now. Again, leave your answers in terms of π.

7. $y = -x + 2,\ x = 0,\ y = 0$



$x = 2 - y$

$\pi\int_0^2 (2-y)^2 dy$

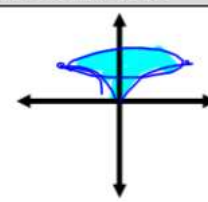
$\pi\int_0^2 (4 - 4y + y^2) dy$

$\pi\left[4y - 2y^2 + \frac{y^3}{3}\right]\Big|_0^2$

$\pi\left[\left(8 - 8 + \frac{8}{3}\right) - 0\right]$

$\boxed{\frac{8}{3}\pi}$

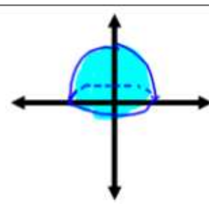
8. $y = \sqrt{x},\ y = 2,$



$y^2 = x$

$\pi\int_0^2 (y^2)^2 dy = \boxed{\frac{32}{5}\pi}$

9. $y = 4 - x^2,\ x = 0,\ y = 0$



$\pm\sqrt{4-y} = x$

$\pi\int_0^4 (\sqrt{4-y})^2 dy$

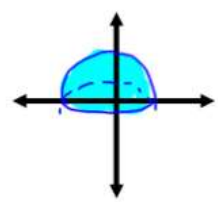
$\pi\int_0^4 (4-y) dy$

$\pi\left[4y - \frac{y^2}{2}\right]\Big|_0^4$

$\pi\left[(16-8) - (0)\right]$

$\boxed{8\pi}$

10. $y = \sqrt{9 - x^2},\ x = 0,\ y = 0$



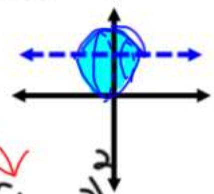
$y^2 = 9 - x^2$

$\pm\sqrt{9 - y^2} = x$

$\pi\int_0^3 \sqrt{9-y^2}^{\,2} dy = \boxed{18\pi}$

Same instructions as above but revolve around the given HORIZONTAL line.

11. $y = 2 - x^2$ and $y = 1$ about the line $y = 1$.



$1 = 2 - x^2$

$x^2 = 1$

$x = \pm 1$

$\pi \int_{-1}^{1} (2 - x^2 - 1)^2 \, dx \qquad (1 - x^2)^2$

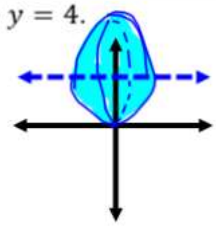
$\pi \int_{-1}^{1} (x^4 - 2x^2 + 1) \, dx$

$\pi \left[\frac{1}{5}x^5 - \frac{2}{3}x^3 + x \right]_{-1}^{1}$

$\pi \left[\left(\frac{1}{5} - \frac{2}{3} + 1 \right) - \left(-\frac{1}{5} + \frac{2}{3} - 1 \right) \right]$

$\boxed{\frac{16}{15}\pi}$

12. $y = x^2$ and $y = 4$ about the line $y = 4$.



$x^2 = 4$

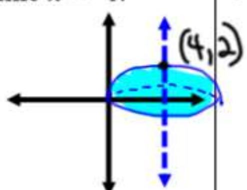
$x = \pm 2$

$\pi \int_{-2}^{2} (4 - x^2)^2 \, dx = \boxed{\frac{512}{15}\pi}$

There are some crazy fractions in this one! Use a calculator to help.

Same instructions as above but revolve around the given VERTICAL line.

13. $y = \sqrt{x}$, $y = 0$, $x = 4$ about the line $x = 4$.



$y^2 = x$

$\pi \int_{0}^{2} (4 - y^2)^2 \, dy$

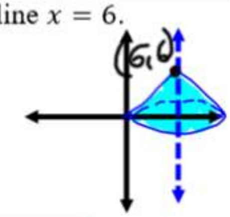
$\pi \int_{0}^{2} (16 - 8y^2 + y^4) \, dy$

$\pi \left[16y - \frac{8}{3}y^3 + \frac{y^5}{5} \right] \Big|_{0}^{2}$

$\pi \left[\left(32 - \frac{64}{3} + \frac{32}{5} \right) - (0) \right]$

$\boxed{\frac{256}{15}\pi}$

14. $y = x$, $y = 0$, $x = 6$ about the line $x = 6$.



$\pi \int_{0}^{6} (6 - y)^2 \, dy = \boxed{72\pi}$

Test Prep: 1D, 2A, 3E, 4A