$\qquad$

## Volume of a Solid with known Cross Sections

$$
V=
$$

where is the
of a cross section perpendicular to the $x$-axis.

| Area of various geometric shapes |  |
| :--- | :--- |
| Square: $\boldsymbol{A}=$ | Equilateral triangle: $\boldsymbol{A}=$ |
| Semicircle: $\boldsymbol{A}=\quad$ | Isosceles right triangle: <br> $\boldsymbol{A}=\quad$ with hypotenuse $\boldsymbol{h}=$ |

1. Find the volume of the solid whose base is bounded by $\boldsymbol{y}=\boldsymbol{x}^{2}$ and $\boldsymbol{y}=\sqrt{\boldsymbol{x}}$, with the indicated cross sections taken perpendicular to the $x$-axis.
a.) Square
b.) Equilateral triangle
c.) Semicircle
d.) Isosceles right triangle (side is the base)
2. Find the volume of the solid whose base is bounded by $y=x^{3}, y=0$, and $x=2$ with cross sections taken perpendicular to the $y$-axis that form a square.


### 11.4 Perpendicular Cross Sections

## Practice

Calculus
The base of an object is bounded by the lines $y=x-4, y=4-x$, and $x=0$. Find the volume of the object with the indicated cross sections taken perpendicular to the $\boldsymbol{x}$-axis. Use a calculator after you set up the integral!

1. Squares
2. Semi-circles
3. Equilateral triangles
4. Isosceles right triangles (side is the base)
5. Set up the integral to find the area of the region bounded by $x=4-y^{2}, y=\frac{2}{3} x-1$, and $x=0$. DO NOT EVALUATE.

With respect to $x$.


With respect to $y$.

The base of an object is bounded by the lines $x^{2}+y^{2}=100$. Find the volume of the object with the indicated cross sections taken perpendicular to the $\boldsymbol{x}$-axis. Use a calculator after you set up the integral!

| 6. Squares | 7. Equilateral triangles |
| :---: | :--- |

8. Semi-circles
9. Isosceles right triangles (hypotenuse = base)
10. The region enclosed by the $y$-axis, the line $y=2$, and the curve $y=\sqrt[3]{x}$ is revolved about the $y$ axis. Set up the integral used to find the volume of the solid that is generated.

The base of an object is bounded by the lines $y=\sqrt{x-1}, x=3$, and $y=0$. Set up the integral to find the volume of the object with the indicated cross sections taken perpendicular to the $y$ axis. DO NOT EVALUATE.
11. Squares $\quad$ 12. Equilateral triangles
15. The region in the first quadrant enclosed by the graphs of $y=2 \ln x, y=2$, and $x=1$ is rotated about the $x$-axis. What is the integral that represents the volume of the resulting solid of revolution?

### 11.4 Perpendicular Cross Sections

Test Prep

1. What is the area of the region in the first quadrant enclosed by the graphs of $y=\sin x, y=2-x$, and the $x$-axis?
(A) 0.552
(B) 0.951
(C) 1.106
(D) 1.600
(E) 2.152

## 2003 Form A \#1 [calculator allowed]

You already did parts (a) and (b) in previous packets. Information needed from parts (a) and (b) is provided.


Let $R$ be the shaded region bounded by the graphs of $y=\sqrt{x}$ and $y=e^{-3 x}$ and the vertical line $x=1$, as shown in the figure above.

Point of intersection: $e^{-3 x}=\sqrt{x}$ at $(0.238734,0.488604)$
(c) The region $R$ is the base of a solid. For this solid, each cross section perpendicular to the $x$-axis is a rectangle whose height is 5 times the length of its base in region $R$. Find the volume of this solid.

