11.4 Perpendicular Cross Sections

Calculus

Practice

The base of an object is bounded by the lines $y = x - 4$, $y = 4 - x$, and $x = 0$. Find the volume of the object with the indicated cross sections taken perpendicular to the x-axis. Use a calculator after you set up the interal!

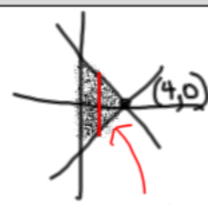
1. Squares

$A = s^2$

$s = 8 - 2x$

$V = \int_0^4 (8-2x)^2\,dx$

$\boxed{V \approx 85.333}$



$4 - x - (x - 4)$

$8 - 2x$

2. Equilateral triangles

$A = \frac{\sqrt{3}}{4} s^2$

$V = \frac{\sqrt{3}}{4} \int_0^4 (8-2x)^2\,dx$

$\boxed{V \approx 36.95}$

3. Semi-circles

$A = \frac{1}{2}\pi r^2$ $\qquad r = \frac{8-2x}{2}$

$V = \int_0^4 \frac{1}{2}\pi \left(\frac{8-2x}{2}\right)^2 dx$

$V = \frac{\pi}{8} \int_0^4 (8-2x)^2\,dx$

$\boxed{V \approx 33.51}$

4. Isosceles right triangles (side is the base)

$A = \frac{1}{2} s^2$ $\qquad s = 8 - 2x$

$V = \frac{1}{2} \int_0^4 (8-2x)^2$

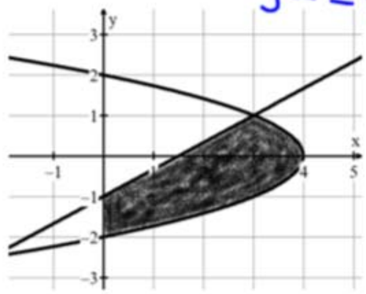
$\boxed{V \approx 42.667}$

5. Set up the integral to find the area of the region bounded by $x = 4 - y^2$, $y = \frac{2}{3}x - 1$, and $x = 0$. DO NOT EVALUATE.

$y = \pm\sqrt{4-x}$ $x = \frac{3}{2}y + \frac{3}{2}$

With respect to x.

$$\int_0^3 \left(\frac{2}{3}x-1\right)-\left(-\sqrt{4-x}\right)dx + \int_3^4 2\sqrt{4-x}\,dx$$



With respect to y.

$$\int_{-2}^{-1} 4-y^2\,dy + \int_{-1}^{1} \frac{5}{2}-y^2-\frac{3}{2}y\,dy$$

The base of an object is bounded by the lines $x^2 + y^2 = 100$. Find the volume of the object with the indicated cross sections taken perpendicular to the x-axis. Use a calculator after you set up the interal!

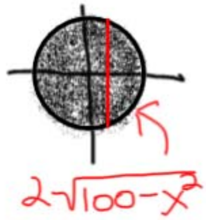
6. Squares

$y = \pm\sqrt{100-x^2}$

$A = s^2$

$s = 2\sqrt{100-x^2}$

$V = \int_{-10}^{10} 400-4x^2\,dx$



$\boxed{V \simeq 5{,}333.333}$

7. Equilateral triangles

$A = \frac{\sqrt{3}}{4}s^2$

$V = \frac{\sqrt{3}}{4}\int_{-10}^{10}(400-4x^2)\,dx$

$\boxed{V \simeq 2309.401}$

8. Semi-circles

$A = \frac{\pi}{2}r^2$ $r = \sqrt{100-x^2}$

$V = \frac{\pi}{2}\int_{-10}^{10} 100-x^2\,dx$

$\boxed{V \simeq 2094.395}$

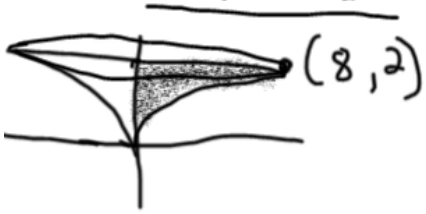
9. Isosceles right triangles (hypotenuse = base)

$A = \frac{h^2}{4}$ $h = 2\sqrt{100-x^2}$

$V = \int_{-10}^{10} \frac{4(100-x^2)}{4}$

$\boxed{V \simeq 1333.333}$

10. The region enclosed by the y-axis, the line $y = 2$, and the curve $y = \sqrt[3]{x}$ is revolved about the y-axis. Set up the integral used to find the volume of the solid that is generated.



$x = y^3$

$\boxed{V = \pi\int_0^2 (y^3)^2\,dy}$

The base of an object is bounded by the lines $y = \sqrt{x-1}$, $x = 3$, and $y = 0$. Set up the integral to find the volume of the object with the indicated cross sections taken perpendicular to the <u>y-axis</u>. DO NOT EVALUATE.

11. Squares

$A = s^2$ $\qquad$ $x = y^2 + 1$

$s = 2 - y^2$

$3 - (y^2 + 1)$

$2 - y^2$

$$\boxed{V = \int_0^{\sqrt{2}} (2-y^2)^2\, dy}$$

12. Equilateral triangles

$A = \frac{\sqrt{3}}{4} s^2$

$$\boxed{V = \frac{\sqrt{3}}{4} \int_0^{\sqrt{2}} (2-y^2)^2\, dy}$$

13. Semi-circles

$A = \frac{\pi}{2} r^2$ $\qquad$ $r = \frac{2-y^2}{2}$

$$V = \int_0^{\sqrt{2}} \frac{\pi}{2} \left(\frac{2-y^2}{2}\right)^2 dy$$

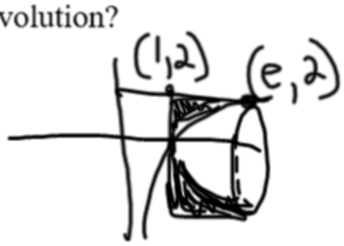
$$\boxed{V = \frac{\pi}{8} \int_0^{\sqrt{2}} (2-y^2)^2\, dy}$$

14. Isosceles right triangles (side is the base)

$A = \frac{1}{2} s^2$

$$\boxed{V = \frac{1}{2} \int_0^{\sqrt{2}} (2-y^2)^2\, dy}$$

15. The region in the first quadrant enclosed by the graphs of $y = 2\ln x$, $y = 2$, and $x = 1$ is rotated about the x-axis. What is the integral that represents the volume of the resulting solid of revolution?



$2 = 2\ln x$

$1 = \ln x$

$e = x$

$$\boxed{V = \pi \int_1^e (2)^2 - (2\ln x)^2\, dx}$$

Test Prep: 1B

2003 Form A #

(c) Length $= \sqrt{x} - e^{-3x}$

Height $= 5\left(\sqrt{x} - e^{-3x}\right)$

Volume $= \int_T^1 5\left(\sqrt{x} - e^{-3x}\right)^2 dx = 1.554$

3 : { 2 : integrand
< −1 > incorrect but has $\sqrt{x} - e^{-3x}$ as a factor
1 : answer }

where $T = 0.238734$