Unit 11 Review – Area and Volume

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you must study all packets from Unit 11.
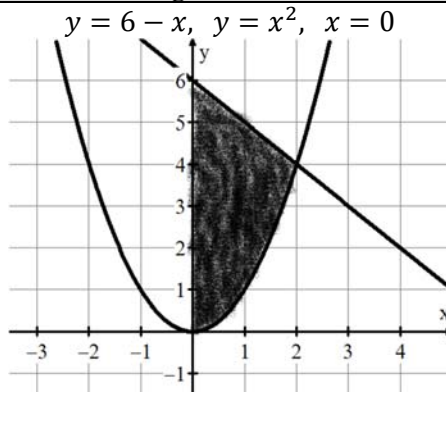
11.1 Area Between Two Curves

$$A = \int_a^b [f(x) - g(x)]\, dx$$

$f(x) \geq g(x)$ for all x in $[a, b]$

Set up the integral to find the area of the shaded region, but DO NOT EVALUATE.

$y = 6 - x, \ y = x^2, \ x = 0$

1. With respect to x.

2. With respect to y.

11.2 Volume – Disc Method

$$V = \pi \int_a^b [R(x)]^2\, dx$$

where $R(x)$ is the "distance" between the axis of revolution and the outside of the object.

Set up the integral to find the volume of the bounded area that revolves about the given line. DO NOT EVALUATE.

3. $y = 4 - x^2$, $x \geq -1$, $y = 0$ about the x-axis.

4. $y = \sqrt{x}$, $y = 0$, $x = 4$ about the line $x = 4$.

11.3 Volume – Washer Method

$$V = \pi \int_a^b [R(x)]^2 - [r(x)]^2 \, dx$$

where $R(x)$ is the radius of the **OUTSIDE** of the object and
$r(x)$ is the radius of the **INSIDE** of the object.

Find the volume of the bounded region when rotating it around the given axis. Round answers to three decimal places.

5. $x = 6 - y^2$ and $y = x - 4$ about the y-axis.	6. $y = x^2 - 2$, $y = -2x + 1$ about the line $y = -3$.

11.4 Perpendicular Cross Sections

$$V = \int_a^b A(x) \, dx$$

where $A(x)$ is the **AREA** of a cross section perpendicular to the x-axis.

Use the area bounded by $y = 2 - x^2$ and $y = 3x + 2$ as the base of a solid with the indicated cross sections. Round answers to 3 decimal places.

7. Equilateral triangles perpendicular to the x-axis $A = \frac{\sqrt{3}}{4} s^2$	8. Semi-circles perpendicular to the y-axis. y-axis!!

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9. The base of a solid is a region in the first quadrant bounded by the x-axis, the y-axis, and the line $x + 3y = 9$, as shown in the figure. If cross sections of the solid perpendicular to the y-axis are isosceles right triangles with the hypotenuses in the xy-plane, what is the volume of the solid?
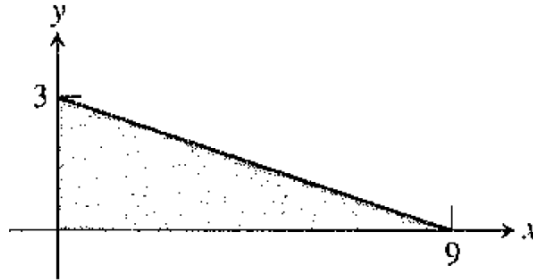
    (A) 6.75   (B) 13.5   (C) 15.188   (D) 20.25   (E) 40.5

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10. The area of the region enclosed by the graph of $y = 5 - x^2$ and the line $y = 1$ is

    (A) $\frac{4}{3}$ (B) $\frac{8}{3}$ (C) 4   (D) $\frac{16}{3}$ (E) $\frac{32}{3}$

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11. If the region enclosed by the graphs of $y = \sqrt{x - 1}, x = 4$ and the x-axis is revolved about the x-axis, the volume of the solid generated is

    (A) $2\pi\sqrt{3}$ (B) $\frac{7\pi}{2}$ (C) 4π (D) $\frac{9\pi}{2}$ (E) 12π

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12. The volume generated by revolving about the x-axis the region above the curve $y = x^3$, below the line $y = 1$, and between $x = 0$ and $x = 1$ is

    (A) $\frac{\pi}{42}$ (B) 0.143π (C) $\frac{\pi}{7}$ (D) 0.643π (E) $\frac{6\pi}{7}$

13. What is the area of the region in the first quadrant enclosed by the graphs of $y = 2 - x^2$, $y = 3\sin x$, and the y-axis?

(A) 0.591   (B) 0.604   (C) 0.982   (D) 1.281   (E) 1.924

14. If the region enclosed by the y-axis and the relation $x = 4 - y^2$ is revolved about the y-axis, the volume of the solid generated is

(A) 25.133   (B) 33.510   (C) 53.617   (D) 107.233   (E) 214.466

15. The base of a solid S is the region enclosed by the graph of $y = \sqrt{\ln(x - 1)}$, the line $x = 2e$, and the x-axis. If the cross sections of S perpendicular to the x-axis are squares, then the volume of S is

(A) 1.587   (B) 3.173   (C) 3.185   (D) 3.501   (E) 6.347

16. If the region enclosed by the y-axis, the curve $y = 4\sqrt{x}$, and the line $y = 8$ is revolved about the x-axis, the volume of the solid generated is

(A) $\frac{32\pi}{3}$ (B) 128π (C) $\frac{128}{3}$ (D) 128 (E) $\frac{128\pi}{3}$