2.1 Average Rate of Change   Name:_____

Recall: Rate of Change

$$= \frac{\rule{3cm}{0.4pt}}{\rule{3cm}{0.4pt}} \quad = \quad \frac{\rule{1.5cm}{0.4pt}}{\rule{1.5cm}{0.4pt}} \quad =$$
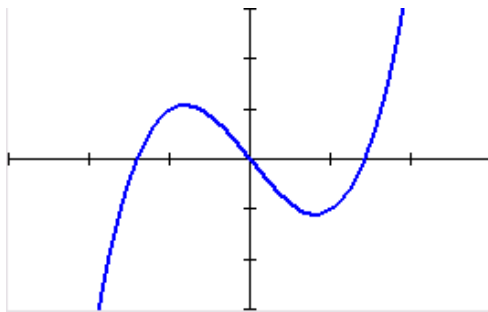
> **Average Rate of Change:**
>   A continuous function $f(x)$ on the interval $[a, b]$ has an average rate of change of
>
> $$\frac{f(b) - f(a)}{b - a} \qquad \text{or} \qquad \frac{f(a) - f(b)}{a - b}$$
>
>   This is also the _____ of the _____ line.

Example 1: Find the average rate of change of $f(x) = x^3 - 2x$ on the interval $\left[\frac{1}{2}, \frac{3}{2}\right]$

Example 2: The function $h(k) = 3k^2 - k$ represents how long it takes Mr. Brust to clean his house where h is measured in hours and k is measured in the number of kids Mr. Brust has. What is the average rate it takes to clean his house if he has between 1 and 4 kids?

Write your questions and thoughts here!

Example 3: Find the average rate of change for $f(x) = x^2 - 4x + 1$ on the interval $[x, x+h]$.

Slope of the Secant Line:

Given a function f, the equation for the slope of the secant line is

$$\frac{f(x+h) - f(x)}{(x+h) - (x)} = \frac{f(x+h) - f(x)}{h}$$

Now summarize what you learned!

_____

_____

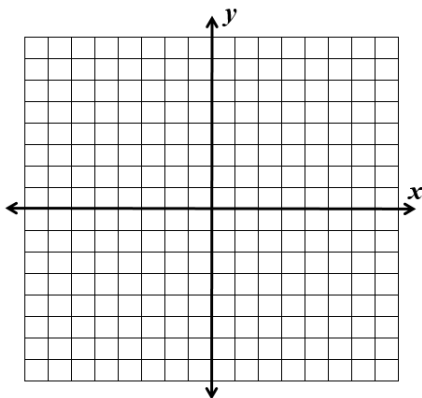_____

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2.1 Average Rate of Change
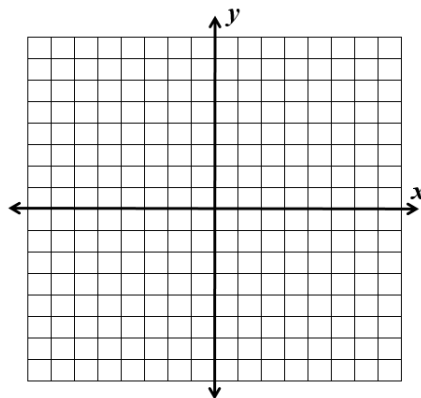Calculus   Name: _____

Practice

Find the average rate of change for each function on the given interval. On the grid provided, sketch the function and draw the secant line.
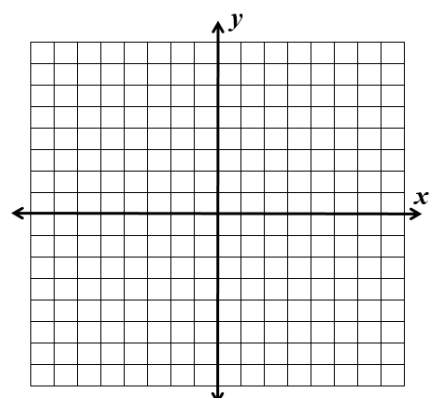
1. $f(x) = x^2 - 2$; $[-1, 3]$	2. $g(x) = 4 - x^2$; $[1, 2]$	3. $h(x) = \sqrt{x+5} + 1$; $[-1, 4]$

Find the average rate of change for each function on the given interval.

4. $g(r) = 2r^2 + r - 1$; $[0,1]$	5. $s(t) = \frac{1}{t-1}$; $[-5,-2]$	6. $a(x) = \ln x$; $[1, e]$

Find the average rate of change for each function on the given interval. Use appropriate units.

7. $s(t) = -t^2 - t + 4$; $[1,5]$ t represents seconds s represents feet	8. $A(t) = 2^t$; $[2,4]$ t represents years A represents dollars	9. $n(m) = \tan m + 4$; $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$ n represents nose hairs m represents months

Find the equation of the secant line on the given interval. Put the equation in slope-intercept form.

10. $v(t) = t^3 - t$; $[-2,2]$	11. $f(x) = \frac{x}{x+2}$; $[-1,1]$	12. $h(t) = \sin t$; $\left[\pi, \frac{3\pi}{2}\right]$

Using the interval $[x, x + h]$, find the expression that represents the slope of the secant line.

13. $f(x) = x^2 - x$

14. $f(x) = \sqrt{x}$

15. $f(x) = 3 - 2x^2$

16. $f(x) = \frac{1}{x}$

2.1 Average Rate of Change

Test Prep

1. The cost of producing x units of a certain item is $c(x) = 2{,}000 + 8.6x + 0.5x^2$. What is the average rate of change of c with respect to x when the level of production increases from $x = 300$ to $x = 310$ units?
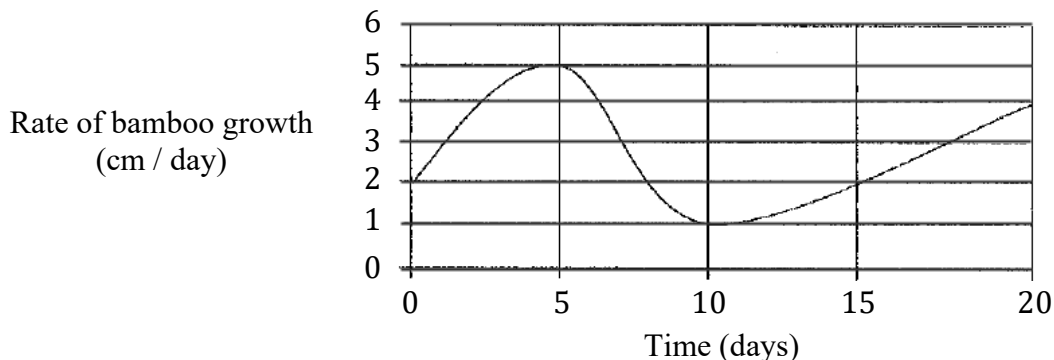
(A) 313.6   (B) 310   (C) 214.2   (D) 200   (E) 10

2. Which of the following is true of the function $f(x) = \sqrt{x^2 + 1}$?

(A) $\lim\limits_{x\to\infty} (f(x) - x) = 0$ and $\lim\limits_{x\to-\infty} (f(x) - x) = 0$ (B) $\lim\limits_{x\to\infty} (f(x) + x) = 0$ and $\lim\limits_{x\to-\infty} (f(x) - x) = 0$

(C) $\lim\limits_{x\to\infty} (f(x) - x) = 0$ and $\lim\limits_{x\to-\infty} (f(x) + x) = 0$ (D) $\lim\limits_{x\to\infty} (f(x) + x) = 0$ and $\lim\limits_{x\to-\infty} (f(x) + x) = 0$

(E) None of the above

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3. Using the graph, what is the average rate of change of $g(t)$ over the interval $0 \le t \le 5$ days?

Rate of bamboo growth
(cm / day)

Time (days)

(A) $\frac{3}{5}$ cm per day per day   (B) 1 cm per day per day   (C) $\frac{7}{5}$ cm per day per day

(D) 3 cm per day per day   (E) $\frac{7}{2}$ cm per day per day

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4. Which of the following functions has a vertical asymptote at $x = 4$?

(A) $\frac{x+5}{x^2-4}$ (B) $\frac{x^2-16}{x-4}$ (C) $\frac{4x}{x+1}$ (D) $\frac{x+6}{x^2-7x+12}$ (E) None of the above

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5. A tank holds 10,000 liters of gasoline. At the bottom of the tank, a lever can be turned to allow the gasoline to be dispensed. The tank can be emptied in exactly 40 minutes. Below is a table which gives the volume v of gasoline (in liters) which remain in the tank after t minutes of draining have taken place.

t (minutes)	0	5	10	15	20	25	30	35	40
v (liters)	4700	4100	3200	2400	2000	1400	800	500	0

During which of the following 10-minute intervals is the average rate of gasoline draining from the tank the least?

(A) $t = 0$ to $t = 10$ minutes   (B) $t = 10$ to $t = 20$ minutes   (C) $t = 15$ to $t = 25$ minutes

(D) $t = 25$ to $t = 35$ minutes   (E) $t = 30$ to $t = 40$ minutes