2.1 Average Rate of Change

Calculus

Name: Solutions

Practice

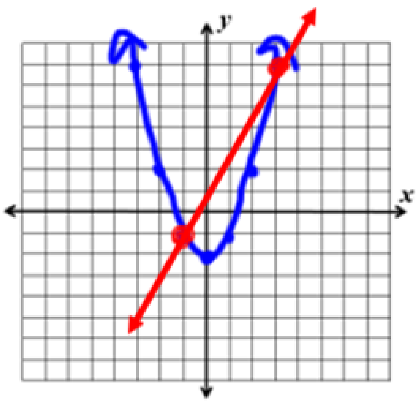
Find the average rate of change for each function on the given interval. On the grid provided, sketch the function and draw the secant line.

1. $f(x) = x^2 - 2;\ [-1, 3]$

$f(-1) = (-1)^2 - 2 = -1$

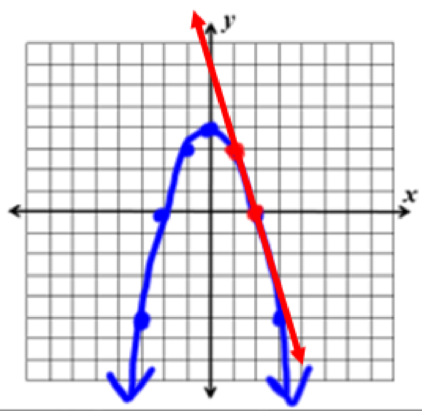
$f(3) = (3)^2 - 2 = 7$

$\frac{7 - -1}{3 - -1} = \frac{8}{4} = \boxed{2}$



2. $g(x) = 4 - x^2;\ [1, 2]$

$\boxed{-3}$

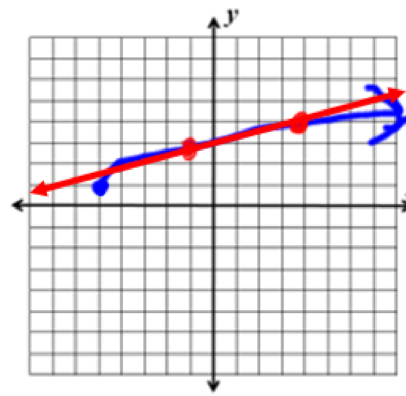


3. $h(x) = \sqrt{x+5} + 1;\ [-1, 4]$

$h(-1) = \sqrt{-1+5} + 1 = 3$

$h(4) = \sqrt{4+5} + 1 = 4$

$\frac{4-3}{4+1} = \boxed{\frac{1}{5}}$



Find the average rate of change for each function on the given interval.

4. $g(r) = 2r^2 + r - 1;\ [0, 1]$

$\boxed{3}$

5. $s(t) = \frac{1}{t-1};\ [-5, -2]$

$s(-5) = \frac{1}{-5-1} = -\frac{1}{6}$

$s(-2) = \frac{1}{-2-1} = -\frac{1}{3}$

$\frac{-\frac{1}{6} - -\frac{1}{3}}{-5 - -2} = \frac{-\frac{1}{6} + \frac{2}{6}}{-3} =$

$\frac{1}{6} \cdot \left(-\frac{1}{3}\right) = \boxed{-\frac{1}{18}}$

6. $a(x) = \ln x;\ [1, e]$

$\boxed{\frac{1}{e-1}}$

Find the average rate of change for each function on the given interval. Use appropriate units.

7. $s(t) = -t^2 - t + 4;\ [1, 5]$

s represents feet

t represents seconds

$s(1) = -1 - 1 + 4 = 2$

$s(5) = -25 - 5 + 4 = -26$

$\frac{-26-2}{5-1} = \frac{-28}{4}$ $\frac{\text{feet}}{\text{seconds}}$

$\boxed{-7 \text{ ft/sec}}$

8. $A(t) = 2^t;\ [2, 4]$

A represents dollars

t represents years

$\boxed{6 \text{ dollars/year}}$

9. $n(m) = \tan m + 4;\ \left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$

n represents nose hairs

m represents months

$n\left(\frac{\pi}{4}\right) = \tan\frac{\pi}{4} + 4 = 5$

$n\left(\frac{3\pi}{4}\right) = \tan\frac{3\pi}{4} + 4 = 3$

$\frac{5-3}{\frac{\pi}{4} - \frac{3\pi}{4}} = \frac{2}{-\frac{\pi}{2}} = 2\left(-\frac{2}{\pi}\right)$

$\boxed{-\frac{4}{\pi} \text{ nose hairs/month}}$

Find the equation of the secant line on the given interval. Put the equation in slope-intercept form.

10. $v(t) = t^3 - t$; $[-2, 2]$

$\boxed{y = 3x}$

11. $f(x) = \frac{x}{x+2}$; $[-1, 1]$

$f(-1) = \frac{-1}{-1+2} = -1$

$f(1) = \frac{1}{3}$

$m = \frac{\frac{1}{3} - -1}{1 - -1} = \frac{\frac{4}{3}}{2} = \frac{2}{3}$

$y + 1 = \frac{2}{3}(x + 1)$

$\boxed{y = \frac{2}{3}x - \frac{1}{3}}$

12. $h(t) = \sin t$; $\left[\pi, \frac{3\pi}{2}\right]$

$\boxed{y = \frac{-2}{\pi}x + 2}$

Using the interval $[x, x+h]$, find the expression that represents the slope of the secant line.

13. $f(x) = x^2 - x$

$\frac{(x+h)^2 - (x+h) - (x^2 - x)}{x + h - x}$

$\frac{x^2 + 2hx + h^2 - x - h - x^2 + x}{h}$

$\frac{h(2x + h - 1)}{h}$

$\boxed{2x + h - 1}$

14. $f(x) = \sqrt{x}$

$\boxed{\frac{1}{\sqrt{x+h} + \sqrt{x}}}$

15. $f(x) = 3 - 2x^2$

$\frac{3 - 2(x+h)^2 - (3 - 2x^2)}{x + h - x}$

$\frac{3 - 2(x^2 + 2hx + h^2) - 3 + 2x^2}{h}$

$\frac{3 - 2x^2 - 4hx - 2h^2 - 3 + 2x^2}{h}$

$\frac{h(-4x - 2h)}{h}$

$\boxed{-4x - 2h}$

16. $f(x) = \frac{1}{x}$

$\boxed{-\frac{1}{x(x+h)}}$

Test Prep: 1A, 2C, 3A, 4D, 5E