2.2 Definition of the Derivative

Calculus

Name: Solutions

Practice

Find the derivative using limits. If the equation is given as $y =$, use Leibniz Notation: $\frac{dy}{dx}$. If the equation is given as $f(x) =$, use Lagrange Notation: $f'(x)$. WRITE SMALL!!

1. $f(x) = 7 - 6x$

$f'(x) = \lim_{h\to 0} \frac{7-6(x+h)-[7-6x]}{h}$

$\lim_{h\to 0} \frac{7-6x-6h-7+6x}{h}$

$\lim_{h\to 0} \frac{-6h}{h}$

$\lim_{h\to 0} -6$

$\boxed{f'(x) = -6}$

2. $y = 5x^2 - x$

$\boxed{\frac{dy}{dx} = 10x - 1}$

3. $y = x^2 + 2x - 9$

$\lim_{h\to 0} \frac{(x+h)^2+2(x+h)-9-[x^2+2x-9]}{h}$

$\lim_{h\to 0} \frac{x^2+2hx+h^2+2x+2h-9-x^2-2x+9}{h}$

$\lim_{h\to 0} \frac{2hx+h^2+2h}{h}$

$\lim_{h\to 0} \frac{\cancel{h}(2x+h+2)}{\cancel{h}}$

$\lim_{h\to 0} 2x+h+2$

$\boxed{\frac{dy}{dx} = 2x + 2}$

4. $y = \sqrt{5x+2}$

$\boxed{\frac{dy}{dx} = \frac{5}{2\sqrt{5x+2}}}$

5. $f(x) = \frac{1}{x-2}$

$\lim_{h\to 0} \frac{\frac{1}{x+h-2} - \frac{1}{x-2}}{h}$

$\lim_{h\to 0} \frac{\frac{x-2-(x+h-2)}{(x+h-2)(x-2)}}{h}$

$\lim_{h\to 0} \frac{-h}{(x+h-2)(x-2)} \cdot \frac{1}{h}$

$\boxed{f'(x) = -\frac{1}{(x-2)^2}}$

For each problem, create an equation of the tangent line of f at the given point. Leave in point-slope.

6. $f(7) = 5$ and $f'(7) = -2$

$\boxed{y-5 = -2(x-7)}$

7. $f(-2) = 3$ and $f'(-2) = 4$

$x_1 \quad y_1 \quad x_1 \quad m$

$\boxed{y-3 = 4(x+2)}$

8. $f(x) = 3x^2 + 2x$;
$f'(x) = 6x + 2$; $x = -2$

$\boxed{y-8 = -10(x+2)}$

9. $f(x) = 10\sqrt{6x+1}$;

$f'(x) = \frac{30}{\sqrt{6x+1}}$; $x = 4$

$f(4) = 10\sqrt{25} = 50$ y_1

$f'(4) = \frac{30}{\sqrt{25}} = 6$ m

$\boxed{y - 50 = 6(x-4)}$

10. $f(x) = \cos 2x$;

$f'(x) = -2\sin 2x$; $x = \frac{\pi}{4}$

$\boxed{y = -2\left(x - \frac{\pi}{4}\right)}$

11. $f(x) = \tan x$;

$f'(x) = \sec^2 x$; $x = \frac{\pi}{3}$

$f\left(\frac{\pi}{3}\right) = \sqrt{3}$ y_1

$f'\left(\frac{\pi}{3}\right) = (2)^2 = 4$ m

$\boxed{y - \sqrt{3} = 4\left(x - \frac{\pi}{3}\right)}$

Identify the original function $f(x)$, and what value of c to evaluate $f'(c)$.

12. $$\lim_{h \to 0} \frac{3(1+h)^2 - 7(1+h) + 1 + (3)}{h}$$

$f(x) = 3x^2 - 7x + 1$

$f'(1)$

13. $$\lim_{h \to 0} \frac{\log(2 - 4(h-5)) - \log(22)}{h}$$

$f(x) = \log(2 - 4x)$

$f'(-5)$

14. $$\lim_{x \to -2} \frac{(3x - 9x^2) + (42)}{x+2}$$

$f(x) = 3x - 9x^2$

$f'(-2)$

15. $$\lim_{x \to 5} \frac{\frac{1}{\sqrt{3x}} - \frac{1}{\sqrt{15}}}{x-5}$$

$f(x) = \frac{1}{\sqrt{3x}}$

$f'(5)$

16. $$\lim_{h \to 0} \frac{e^{6(3+h)+1} - e^{19}}{h}$$

$f(x) = e^{6x+1}$

$f'(3)$

17. $$\lim_{x \to \frac{\pi}{2}} \frac{6x^2 \sin x - \frac{3}{2}\pi^2}{x - \frac{\pi}{2}}$$

$f(x) = 6x^2 \sin x$

$f'\left(\frac{\pi}{2}\right)$

For each problem, use the information given to identify the meaning of the two equations in the context of the problem. Write in full sentences!

18. C is the number of championships Sully has won while coaching basketball.
t is the number of years since 2002 for the function $C(t)$.
$C(12) = 3$ and $C'(12) = 0.4$

By 2014, Sully won 3 championships.

In 2014, the rate at which Sully is winning championships is increasing by 0.4 championships per year.

19. d is the distance (in miles) from home when you walk to school.
h is the number of hours since 7:00 a.m. for the function $d(h)$.
$d(0.2) = 0.5$ and $d'(0.2) = -11$

At 7:12 a.m. you are 0.5 miles from home.

At 7:12 a.m. you are going back home at a rate of 11 mph.

20. W is the number of cartoon shows Mr. Kelly watches every week.
x is the number of children Mr. Kelly has for the function $W(x)$.
$W(7) = 25$ and $W'(7) = 3$

If Mr. Kelly has 7 kids, he watches 25 cartoons every week.

If he has 7 kids, the rate of watching cartoons is increasing by 3 per week.

21. g is the number of gray hairs on Mr. Brust's head.
x is the number of students in his 4^{th} period.
$g(26) = 501$ and $g'(15) = 130$

With 26 kids in his 4th period, Mr. Brust has 501 gray hairs.

With 15 kids in his 4th period, Mr. Brust is gaining 130 gray hairs per kid.

Test Prep: 1D, 2C, 3D

Hint for Test Prep #2. To find $f'(x)$, write out the definition of the derivative, which is exactly what $f'(x)$ equals. Then, use the first part of the question, $f(x+y)$, to break apart that definition.