Calculus

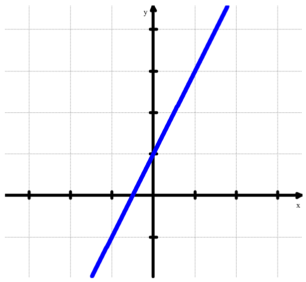
Write your questions and thoughts here!

2.3 Differentiability

Name:___________________________

Notes

A graph of a function is shown below. Write down its equation on line #1.



1. $y =$ ____________________
2. $y =$ ____________________
3. $y =$ ____________________

Differentiability:

The derivative exists for each point in the domain. The graph must be a smooth line or curve for the derivative to exist. In other words, the graph looks like a line if you zoom in (_____________________).

The derivative ______________________ where the function has a

1.

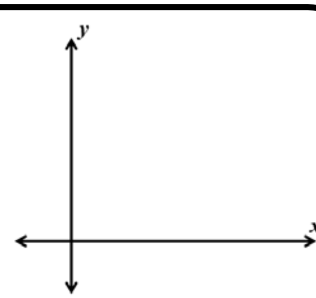
2.

3.

True or **False**	**True** or **False**
Differentiability implies continuity.	Continuity implies differentiability.

Mean Value Theorem:

If a function f is differentiable (and thereby continuous) over the interval ________, then there exists a point ___ within that open interval where the instantaneous rate of change equals the average rate of change over the interval.



2.3 Differentiability

Notes

Write your questions and thoughts here!

Given $f(x)$ and $f'(x)$ on a given interval $[a, b]$, find a value c that satisfies the Mean Value Theorem.

1. $f(x) = -2x^2 + 16x - 26;\ \ 4 \le x \le 6$
   $f'(x) = -4x + 16$

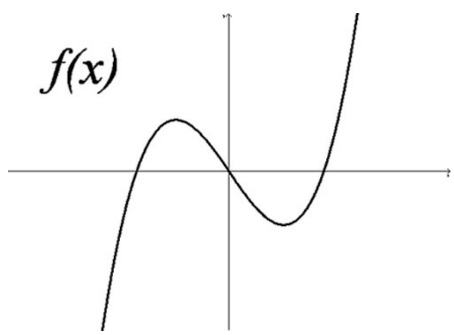
Derivatives and Calculators:

Using the calculator to find the value of the derivative at a point.

2. Find the value of $f'(0.57)$ if $f(x) = \frac{x^3}{\ln x}$

Graph of a function f and its derivative f'

Focus on the __________ of f. The __________ of f is the __________ of f'.



Now summarize what you learned!

2.3 Differentiability

Calculus   Name: ______________________________   **Practice**

Given $f(x)$ and $f'(x)$ on a given interval $[a,b]$, find a value c that satisfies the Mean Value Theorem.

1. $f(x) = -x^2 + 4x - 2;\ [-1, 2]$
   $f'(x) = -2x + 4$

2. $f(x) = \frac{x^2}{2} + 4x + 7;\ [-7, -3]$
   $f'(x) = x + 4$

3. $f(x) = -2x^2 + 12x - 15;$
   $[2, 4]$
   $f'(x) = -4x + 12$

4. $f(x) = x^3 - 12x;\ [-2, 2]$
   $f'(x) = 3x^2 - 12$

5. $f(x) = \sin(2x);\ \left[\frac{\pi}{6}, \frac{\pi}{3}\right]$
   $f'(x) = 2\cos(2x)$

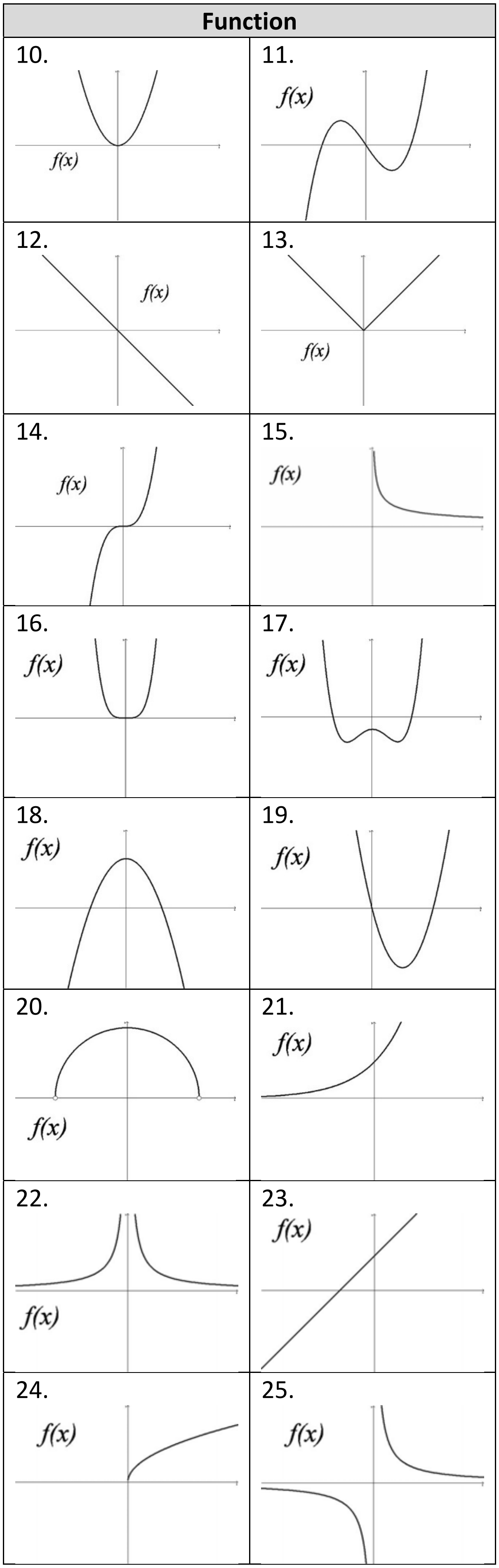
6. $f(x) = x^3 + 24x - 16;\ [0, 4]$
   $f'(x) = 3x^2 + 24$

7. $f(x) = \sqrt{9 - x^2};\quad [0, 3]$
   $f'(x) = \frac{-2x}{\sqrt{9-x^2}}$

8. $f(x) = \sin x + \cos x;\ [0, 2\pi]$
   $f'(x) = \cos x - \sin x$

9. $f(x) = (x^2 - 2x)e^x;\ [0, 2]$
   $f'(x) = (x^2 - 2)e^x$

Match each function with the graph of its derivative.



10. ______

11. ______

12. ______

13. ______

14. ______

15. ______

16. ______

17. ______

18. ______

19. ______

20. ______

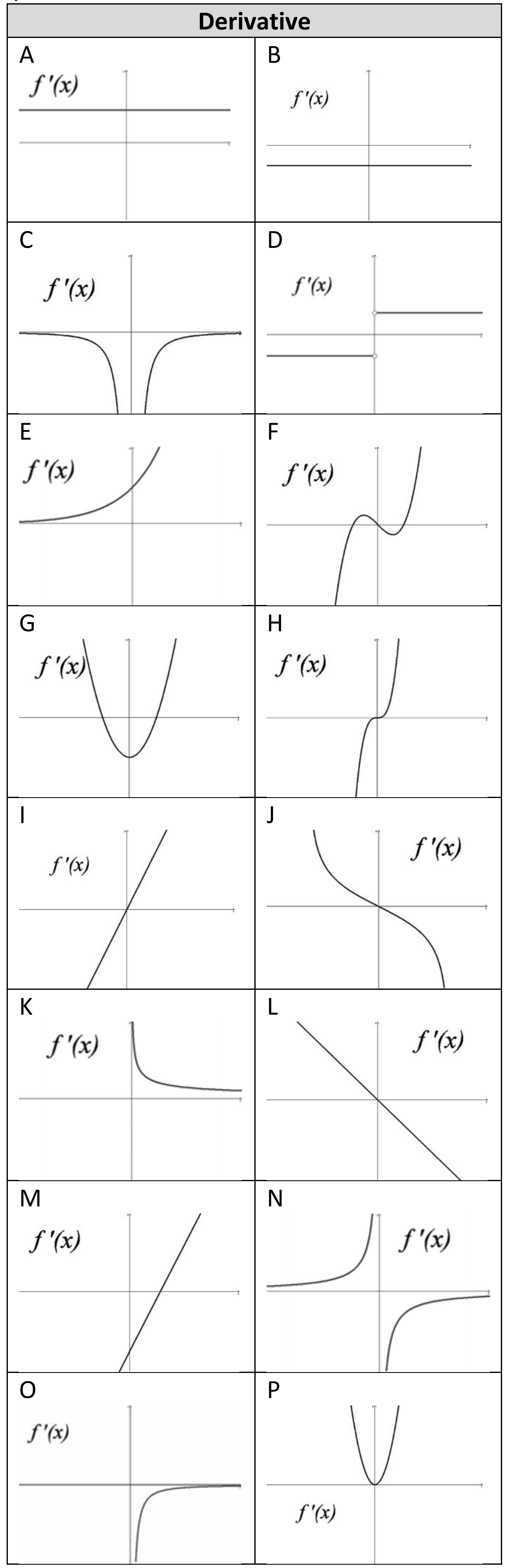
21. ______

22. ______

23. ______

24. ______

25. ______



Using a calculator find the value of the derivative at a given point. DON'T show any work. You should be able to quickly find the answer with a calculator.

26. $f(x) = x^2 + 5x$ $f'(1.98) =$	27. $f(x) = \csc 5x$ $f'\left(\frac{\pi}{2}\right) =$	28. $f(x) = \ln x$ $f'(205) =$
29. $f(x) = \frac{1}{x}$ $f'(\sqrt{2}) =$	30. $f(x) = e^{7x}$ $f'(1.5) =$	31. $f(x) = 8x^2 - 5x^3$ $f'\left(\frac{1}{3}\right) =$

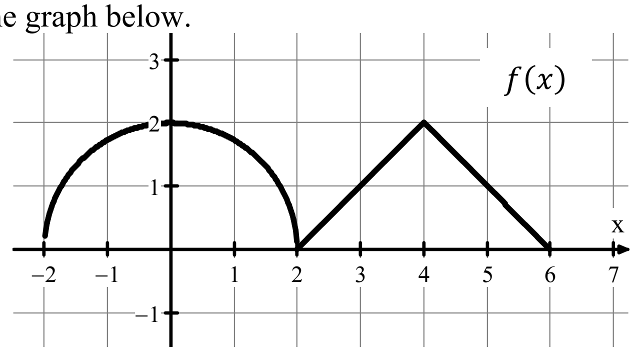
Test Prep

2.3 Differentiability

1. f is continuous for $a \le x \le b$ but not differentiable for some c such that $a < c < b$. Which of the following could be true?

(A) $x = c$ is a vertical asymptote of the graph of f.
(B) $\lim_{x \to c} f(x) \neq f(c)$
(C) The graph of f has a cusp at $x = c$.
(D) $f(c)$ is undefined.
(E) None of the above

Questions 2 and 3 refer to the graph below.



2. The graph of $f(x)$, shown above, consists of a semicircle and two line segments. $f'(1) =$

(A) -1
(B) $-\frac{1}{\sqrt{3}}$
(C) $\frac{1}{\sqrt{3}}$
(D) 1
(E) $\sqrt{3}$

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3. For which values of x does $f'(x) = 0$?

(A) 0 only
(B) 2 only
(C) 0 and 4 only
(D) $-2, 2,$ and 6 only
(E) $-2, 0, 2, 4,$ and 6

4. If f is a differentiable function and $f(0) = -1$ and $f(4) = 3$, then which of the following must be true?

I. There exists a c in $[0,4]$ where $f(c) = 0$.

II. There exists a c in $[0,4]$ where $f'(c) = 0$.

III. There exists a c in $[0,4]$ where $f'(c) = 1$.

(A) I only (B) II only (C) I and II only

(D) I and III only (E) I, II, and III

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5. If $f'(x) = \tan^{-1}(x^3 - x)$, at how many points is the tangent line to the graph of $y = f(x)$ parallel to the line $y = 2x$?

(A) None (B) One (C) Two (D) Three (E) Infinitely many

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6. $\lim_{x \to 0} \frac{\sin^3(3x)}{x^3} =$

(A) 0 (B) 1 (C) 3 (D) 9 (E) 27