2.3 Differentiability

Calculus Name: Solutions **Practice**

Given $f(x)$ and $f'(x)$ on a given interval $[a,b]$, find a value c that satisfies the Mean Value Theorem.

1. $f(x) = -x^2 + 4x - 2;\ [-1,2]$
 $f'(x) = -2x + 4$

$f(-1) = -(1) - 4 - 2 = -7$
$f(2) = -4 + 8 - 2 = 2$
$\frac{-7-2}{-1-2} = \frac{-9}{-3} = 3$
$f'(c) = 3$
$-2c + 4 = 3$
$-2c = -1$
$\boxed{c = \frac{1}{2}}$

2. $f(x) = \frac{x^2}{2} + 4x + 7;\ [-7,-3]$
 $f'(x) = x + 4$

$\boxed{c = -5}$

3. $f(x) = -2x^2 + 12x - 15;$
 $[2,4]$
 $f'(x) = -4x + 12$

$f(2) = -8 + 24 - 15 = 1$
$f(4) = -32 + 48 - 15 = 1$
$\frac{1-1}{4-2} = \frac{0}{2} = 0$
$f'(c) = 0$
$-4c + 12 = 0$
$\boxed{c = 3}$

4. $f(x) = x^3 - 12x;\ [-2,2]$
 $f'(x) = 3x^2 - 12$

$\boxed{c = \pm\frac{2}{\sqrt{3}} \text{ or } \pm\frac{2\sqrt{3}}{3}}$

5. $f(x) = \sin(2x);\ \left[\frac{\pi}{6}, \frac{\pi}{3}\right]$
 $f'(x) = 2\cos(2x)$

$f(\pi/6) = \sin(\pi/3) = \frac{\sqrt{3}}{2}$
$f(\pi/3) = \sin(2\pi/3) = \frac{\sqrt{3}}{2}$
$\frac{\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}}{\pi/3 - \pi/6} = \frac{0}{\pi/6} = 0$
$2\cos(2c) = 0$
$\cos(2c) = 0$
$2c = \pi/2$ or $2c = 3\pi/2$
$\boxed{c = \pi/4}$ or $\cancel{c = 3\pi/4}$

Not in the interval.

6. $f(x) = x^3 + 24x - 16;\ [0,4]$
 $f'(x) = 3x^2 + 24$

$\boxed{c = \frac{4}{\sqrt{3}}}$

The negative answer is not in the interval.

7. $f(x) = \sqrt{9 - x^2};\ \ [0,3]$
 $f'(x) = \frac{-2x}{\sqrt{9-x^2}}$

$f(0) = 3$
$f(3) = 0$
$\frac{3-0}{0-3} = \frac{3}{-3} = -1$
$\frac{-2x}{\sqrt{9-x^2}} = -1$
$-2x = -\sqrt{9-x^2}$
$4x^2 = 9 - x^2$
$5x^2 = 9$
$\boxed{c = \frac{3}{\sqrt{5}}}$

The negative answer is not in the interval.

8. $f(x) = \sin x + \cos x;\ [0, 2\pi]$
 $f'(x) = \cos x - \sin x$

$f(0) = 1$
$f(2\pi) = 1$
$\frac{1-1}{2\pi - 0} = 0$
$\cos x - \sin x = 0$
$\cos x = \sin x$

When are the cosine and sine values the same on the unit circle?

$\boxed{c = \pi/4,\ 5\pi/4}$

9. $f(x) = (x^2 - 2x)e^x;\ [0,2]$
 $f'(x) = (x^2 - 2)e^x$

$f(0) = 0$
$f(2) = 0$
$\frac{0-0}{2-0} = 0$
$(x^2 - 2)e^x = 0$
$x^2 - 2 = 0$ or $e^x = 0$
$\boxed{x = \sqrt{2}}$

Not possible

The negative answer is not in the interval.

Matching Graphs: 10-I, 11-G, 12-B, 13-D, 14-P, 15-O, 16-H, 17-F, 18-L, 19-M, 20-J, 21-E, 22-N, 23-A, 24-K, 25-C.

Using a calculator find the value of the derivative at a given point. DON'T show any work. You should be able to quickly find the answer with a calculator.		
26. $f(x) = x^2 + 5x$ $f'(1.98) =$ 8.96	27. $f(x) = \csc 5x$ $f'\left(\frac{\pi}{2}\right) =$ 0	28. $f(x) = \ln x$ $f'(205) =$ 0.005
29. $f(x) = \frac{1}{x}$ $f'(\sqrt{2}) =$ -0.5	30. $f(x) = e^{7x}$ $f'(1.5) =$ 254,210.595	31. $f(x) = 8x^2 - 5x^3$ $f'\left(\frac{1}{3}\right) =$ 3.667

Test Prep: 1C, 2B, 3A, 4D, 5A, 6E