Name: Solutions Date:_______________ Period: _____ **Review**

2 Review – The Derivative

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you must study all packets from Unit 2.

2.1 Average Rate of Change

A continuous function $f(x)$ on the interval $[\boldsymbol{a}, \boldsymbol{b}]$ has an average rate of change of

$$\frac{f(b)-f(a)}{b-a} \quad \text{or} \quad \frac{f(a)-f(b)}{a-b}$$

This is also the **SLOPE** of the **TANGENT** line.

Find the average rate of change for each function on the given interval. Use units when necessary.

1. $w(t) = 5t^2 - 5t + 1; [-2, 1]$

$w(-2) = 31$

$w(1) = 1$

$\frac{31-1}{-2-1} = \frac{30}{-3} =$

-10

2. $s(x) = \frac{x+5}{3};\ [1, 7]$

$s(1) = 2$

$s(7) = 4$

$\frac{2-4}{1-7} = \frac{-2}{-6}$

$1/3$

3. $B(t) = \cos\left(\frac{\pi}{3}t\right);\ \left[\frac{3}{2}, 6\right]$
B represents wild boar
t represents weeks

$B(\frac{3}{2}) = \cos(\frac{\pi}{2}) = 0$

$B(6) = \cos(2\pi) = 1$

$\frac{0-1}{3/2-6} = \frac{-1}{-9/2} = 2/9$

$2/9$ boar per week

2.2 Definition of the Derivative

<u>Defintion of the derivative:</u>

This limit gives an expression that calculates the *instantaneous* rate of change (slope of the tangent line) of $f(x)$ at any given x-value.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

<u>Derivative at a point:</u>

Finding the derivative at a specific x-value. We will call this value c.

$$f'(c) = \lim_{h\to 0} \frac{f(c+h)-f(c)}{h}$$

or

$$f'(c) = \lim_{x\to c} \frac{f(x)-f(c)}{x-c}$$

Find the derivative using limits. WRITE SMALL!!	Create an equation of the tangent line of f at the given point. Leave in point-slope.
4. $y = 2x^2 + 3x - 1$	5. $f(x) = -2x^3 + 3x$; $f'(x) = -6x^2 + 3$; $x = -1$

$$y' = \lim_{h\to 0} \frac{2(x+h)^2 + 3(x+h) - 1 - [2x^2 + 3x - 1]}{h}$$

$$\frac{2x^2 + 4xh + h^2 + 3x + 3h - 1 - 2x^2 - 3x + 1}{h}$$

$$y' = \lim_{h\to 0} \frac{h(4x + h + 3)}{h}$$

$$y' = 4x + 3$$

$f(-1) = -1$

$f'(-1) = -3$

$$y + 1 = -3(x+1)$$

Identify the original function $f(x)$, and what value of c to evaluate $f'(c)$.

6. $\lim_{h\to 0} \frac{-(3+h)^2 + (3+h) - 4 + (10)}{h}$

$f(x) =$ $-x^2 + x - 4$

$c =$ 3

7. $\lim_{x\to 5} \frac{(4x - 2x^3) + (130)}{x - 5}$

$f(x) =$ $4x - 2x^3$

$c =$ 5

2.3 Differentiability

8. When does the derivative fail to exist?

Discontinuity, corner or cusp, vertical tangent.

9. What is the difference between the Mean Value Theorem and the Intermediate Value Theorem?

The MVT states focuses on the rate of change (slope) of the function, while the IVT focuses on the value (y-value) of the function.

Given $f(x)$ and $f'(x)$ on a given interval $[a, b]$, find a value c that satisfies the Mean Value Theorem.	Using a calculator find the value of the derivative at a given point.
10. $f(x) = 4x^2 - 3x + 5$; $[-2, 2]$; $f'(x) = 8x - 3$	11. $f(x) = 0.2 \ln x$; $f'(0.7) =$

$f(-2) = 27$

$f(2) = 15$

$$\frac{27 - 15}{-2 - 2} = -3$$

$8x - 3 = -3$

$x = 0$

0.2857

Check your 2.3 packet on matching graphs between f and f'.