Find the derivative of the following.

1. $f(x) = 2x^3 - 4x + 5$

   $f'(x) = 6x^2 - 4$

2. $y = 3x^{100} - 2x^8 - 7x$

   $y' = 300x^{99} - 16x^7 - 7$

3. $g(x) = 5x^{-2} - \frac{1}{2}x^4$

   $g'(x) = -10x^{-3} - 2x^3$

   $g'(x) = \frac{-10}{x^3} - 2x^3$

4. $h(x) = \frac{x^6}{3} + 6x^{2/3} - 4x^{1/2} + 2$

   $h'(x) = 2x^5 + \frac{4}{\sqrt[3]{x}} - \frac{2}{\sqrt{x}}$

5. $f(x) = \frac{1}{x^3} + \frac{12}{x}$

   $f(x) = x^{-3} + 12x^{-1}$

   $f'(x) = -3x^{-4} - 12x^{-2}$

   $f'(x) = \frac{-3}{x^4} - \frac{12}{x^2}$

6. $y = \frac{3}{x^{-2}} - \frac{1}{(6x)^2}$

   $\frac{dy}{dx} = 6x + \frac{1}{18x^3}$

7. $f(x) = \sqrt{x} + 3\sqrt[3]{x} + 2$

   $f(x) = x^{1/2} + 3x^{1/3} + 2$

   $f'(x) = \frac{1}{2}x^{-1/2} + x^{-2/3}$

   $f'(x) = \frac{1}{2\sqrt{x}} + \frac{1}{\sqrt[3]{x^2}}$

8. $y = \sqrt[3]{x^2} + 8\sqrt[4]{x^7}$

   $\frac{dy}{dx} = \frac{2}{3\sqrt[3]{x}} + 14\sqrt[4]{x^3}$

9. $f(x) = \frac{1}{\sqrt{x}} + \frac{3}{6x}$

   $f(x) = x^{-1/2} + \frac{3}{6}x^{-1}$

   $f'(x) = -\frac{1}{2}x^{-3/2} - \frac{1}{2}x^{-2}$

   $f'(x) = -\frac{1}{2\sqrt{x^3}} - \frac{1}{2x^2}$

10. $f(x) = \frac{1}{\sqrt{x}} + \frac{3}{\sqrt[5]{x^2}}$

    $f'(x) = \frac{-1}{2\sqrt{x^3}} - \frac{6}{5\sqrt[5]{x^7}}$

11. $s(t) = -16t^2 + 40t + 5$

    $s'(t) = -32t + 40$

12. $y = \pi x^2 - \pi$

    $y' = 2\pi x$

13. $V(r) = \frac{4}{3}\pi r^3$

    $V'(r) = 4\pi r^2$

14. $f(x) = \frac{2x^3 + 4x - 5}{x}$

    $f'(x) = 4x + \frac{5}{x^2}$

15. $g(x) = \frac{6x^3 + 4x^2 - 9x}{3}$

    $g(x) = \frac{6x^3}{3} + \frac{4x^2}{3} - \frac{9x}{3}$

    $g(x) = 2x^3 + \frac{4}{3}x^2 - 3x$

    $g'(x) = 6x^2 + \frac{8}{3}x - 3$

Find the derivatives of the following.

16. $f(x) = 3x^7 - 4x^3 + 5x$

    $f'(x) = 21x^6 - 12x^2 + 5$

    $f''(x) = 126x^5 - 24x$

    $f'''(x) = 630x^4 - 24$

    $f^{(4)}(x) = 2520x^3$

17. $y = 4\sqrt{x} + e$ $\quad y = 4x^{1/2} + e$

    $\frac{dy}{dx} = 2x^{-1/2} = \frac{2}{\sqrt{x}}$

    $\frac{d^2y}{dx^2} = -1x^{-3/2} = -\frac{1}{\sqrt{x^3}}$

18. $y = \frac{1}{x^3} - \frac{1}{2}x^4 + ex^2$

    $y' = -\frac{3}{x^4} - 2x^3 + 2ex$

    $y'' = \frac{12}{x^5} - 6x^2 + 2e$

    $y''' = -\frac{60}{x^6} - 12x$

Given $f(x) = 3x^2 - x + 2$, $g(x) = \frac{1}{x^3} + e^2$, and $h(x) = \sqrt{x}$, find the following.

19. $f'(2) = 11$

$f'(x) = 6x - 1$

$f'(2) = 6(2) - 1$

$f'(2) = 11$

20. $g'''(-3) = -\frac{20}{243}$

21. $2h''(4) = -\frac{1}{16}$

$h(x) = x^{1/2}$

$h'(x) = \frac{1}{2}x^{-1/2}$

$h''(x) = -\frac{1}{4}x^{-3/2}$

$h''(4) = \frac{-1}{4\sqrt{\ }} = \frac{-1}{4(8)} = \frac{-1}{32}$

$2\left(-\frac{1}{32}\right) = -\frac{2}{32} = \frac{-1}{16}$

22. Find the slope of $f(x)$ at $x = 3$.

17

23. At what value of x is $f'(x) = 0$?

$f'(x) = 6x - 1$

$0 = 6x - 1$

$+1 \quad +1$

$\frac{1}{6} = \frac{6x}{6}$

$\frac{1}{6} = x$

24. What is the slope of the tangent line of $h(x)$ at the point (16, 4) ?

$\frac{1}{8}$

Find the slope of the tangent line.

25. $f(x) = 2\sqrt{x} - \pi^2$

$f(x) = 2x^{1/2} - \pi^2$

$f'(x) = x^{-1/2}$

$f'(x) = \frac{1}{\sqrt{x}}$

26. $y = -2x^3 + \frac{1}{2}x^2 - 7x + 5$

$\frac{dy}{dx} = -6x^2 + x - 7$

27. $g(x) = \frac{1}{x^2} - \frac{1}{2x}$

$g(x) = x^{-2} - \frac{1}{2}x^{-1}$

$g'(x) = -2x^{-3} + \frac{1}{2}x^{-2}$

$g'(x) = -\frac{2}{x^3} + \frac{1}{2x^2}$

Is the slope of the tangent line positive, negative, or zero at the given point?

28. $f(x) = \frac{4x^3 - 16x^2}{2x}$ at $x = 2$

zero

29. $y = 2x^4 + 5x^3$ at $x = -2$

$y' = 8x^3 + 15x^2$

$8(-2)^3 + 15(-2)^2$

$-64 + 60$

-4

negative

30. $g(x) = 3\sqrt[3]{x^5} - 4x^{-1}$ at $x = 8$

positive

Write the equation of the tangent line and the normal line that point given.

31. $f(x) = 3\sqrt{x} + 4$ at $x = 4$

$f(x) = 3x^{1/2} + 4$

$f'(x) = \frac{3}{2}x^{-1/2} = \frac{3}{2\sqrt{x}}$

$f'(4) = \frac{3}{2\sqrt{4}} = \frac{3}{4}$

$f(4) = 3\sqrt{4} + 4 = 10$

$f(4) = 10$

$(4, 10)$

Tangent Line

$y - 10 = \frac{3}{4}(x - 4)$

or

$y = \frac{3}{4}x + 7$

Normal Line

$y - 10 = \frac{-4}{3}(x - 4)$

or

$y = \frac{-4}{3}x + \frac{46}{3}$

32. $y = \frac{x^2 + 3x - 4}{2}$ at $x = 8$

Tangent Line

$y - 42 = \frac{19}{2}(x - 8)$

or

$y = \frac{19}{2}x - 34$

Normal Line

$y - 42 = -\frac{2}{19}(x - 8)$

or

$y = -\frac{2}{19}x + \frac{814}{19}$

The function is graphed below. Write the equation of the tangent line at the given point and graph it.

33. $f(x) = -x^2 - 2x - 1$ at $x = -2$

$f'(x) = -2x - 2$

$f'(-2) = -2(-2) - 2$

$f'(-2) = 2$

$f(-2) = -(-2)^2 - 2(-2) - 1$

$f(-2) = -1$

$\boxed{y + 1 = 2(x + 2)}$

or

$y = 2x + 3$

34. $y = \frac{x^3}{2} - \frac{x^2}{2} - x$ at $x = 1$

$\boxed{y + 1 = -\frac{1}{2}(x - 1)}$

or

$y = -\frac{1}{2}x - \frac{1}{2}$

The function is graphed below. Write the equation of the normal line at the given point and graph it.

35. $f(x) = -x^3 + 2x^2 - 2$ at $x = 2$

$f'(x) = -3x^2 + 4x$

$f'(2) = -3(2)^2 + 4(2)$

$f'(2) = -4$

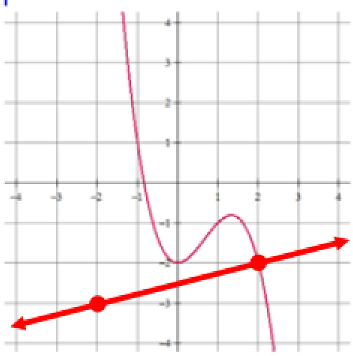
$f(2) = -(2)^3 + 2(2)^2 - 2$

$f(2) = -2$

$\boxed{y + 2 = \frac{1}{4}(x - 2)}$

or

$y = \frac{1}{4}x - \frac{5}{2}$



36. $y = x^2 + 6x + 9$ at $x = -2$

$\boxed{y - 1 = -\frac{1}{2}(x + 2)}$

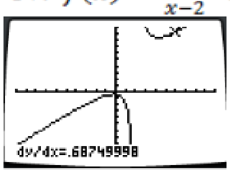
or

$y = -\frac{1}{2}x$

You will need to use a graphing calculator for 37-42

Use the graph to find the derivative of the function at the given value. Round to nearest thousandth.

37. $f(x) = \frac{x^2+1}{x-2}$ at $x = 6$

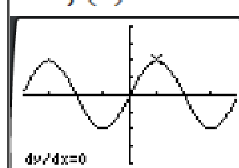


$f'(6) = 0.687$

38. $y = e^x$ at $x = -1$

0.368

39. $f(\theta) = 2\sin\theta$ at $\theta = \frac{\pi}{2}$



ians! ZOOM, TRIG

w $-2\pi < \theta < 2\pi$

$f'(\frac{\pi}{2}) = 0$

Write the equation of the tangent line at the point given and sketch the graph. Round to nearest thousandth.

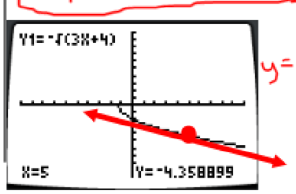
40. $f(x) = -\sqrt{3x+4}$ at $x = 5$

$f'(5) = -0.344$

$\boxed{y + 4.359 = -0.344(x - 5)}$

or

$y = -0.344x - 2.639$



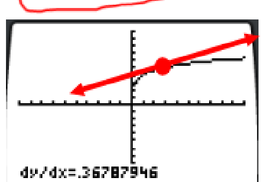
41. $y = \ln(x) + 4$ at $x = e$

0.368

$\boxed{y - 5 = 0.368(x - 2.718)}$

or

$y = 0.368x + 4$



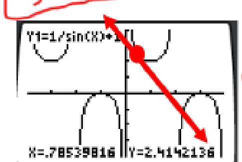
42. $f(\theta) = \csc\theta + 1$ at $\theta = \frac{\pi}{4}$

$f'(\frac{\pi}{4}) = -1.414$

$\boxed{y - 2.414 = -1.414(x - 0.785)}$

or

$y = -1.414x + 3.525$



MULTIPLE CHOICE

1. A
2. D
3. C
4. A
5. A

FREE RESPONSE

Your score: _____ out of 5

Use the table to answer the questions below.

x	$f(x)$	$f'(x)$	$f''(x)$	$g(x)$	$g'(x)$	$g''(x)$
0	-10	1	2	-7	3	-4
2	-4	5	2	-1	7	8
5	20	11	2	83	58	26

1. Find the average rate of change of f over the interval $0 \le x \le 2$. Find the value of x at which the instantaneous velocity of g is equal to the average rate of change of f over the interval $0 \le x \le 2$.

average rate of change of *f*

$\frac{f(2)-f(0)}{2-0} = \frac{-4-(-10)}{2-0} = \frac{6}{2} = 3 \longleftarrow$ **(1 point)**

instantaneous rate of change of *g*

$g'(x) = 3$ when $x = 0 \longleftarrow$ **(1 point)**

2. Write the equation of the line normal to $g(x)$ at the point where $x = 2$.

(1 point) $\longrightarrow$ $y + 1 = -\frac{1}{7}(x-2)$ or $y = -\frac{1}{7}x - \frac{5}{7}$

3. $(f+g)''(5) =$

$f''(5) + g''(5) = 2 + 26 = 28 \longleftarrow$ **(1 point)**

4. If $B = f(x) - 2g(x)$, then $B'(0) =$

$B'(0) = f'(0) - 2g'(0) = 1 - 2(3) = -5 \longleftarrow$ **(1 point)**