3.3 Velocity & other Rates of Change

NOTES

CALCULUS

Write your questions here!

Verbally and Algebraically

The area of circle whose radius is measured in inches.

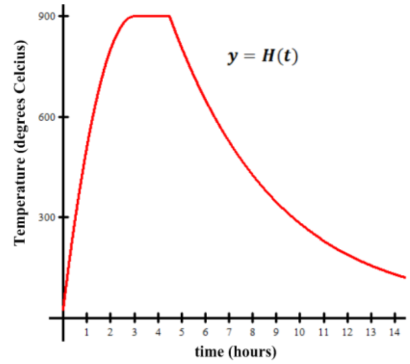
$A(4) =$

$A'(4) =$

Average rate of change from 3 to 5

Graphically

Temperature of a kiln $y = H(t)$



Compare

$H'(2) =$ $H'(4) =$ $H'(8) =$

Average rate of change 0 to 3

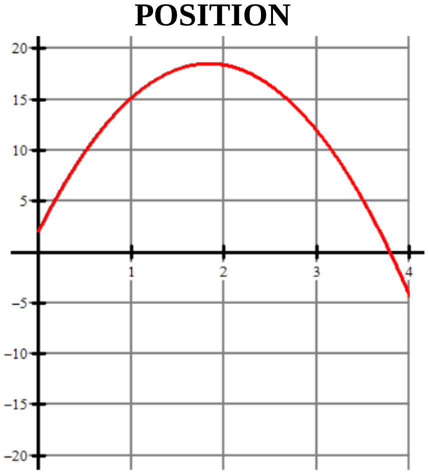
Calculators Produced (in hundreds)	6	7.5	8	10	11	12
Profit (in thousands of dollars)	14	12.8	11	13.2	14.2	15.3

Approximate $P'(9)$

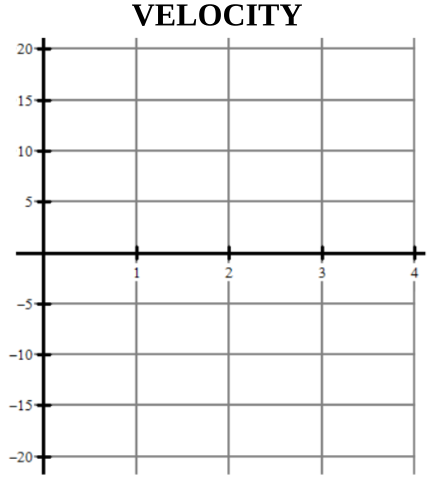
Velocity, Speed, and Acceleration

Mr Bean is playing catch with his best friend, himself. He throws a tennis ball straight up into the air. The height of the ball is modeled by $s(t) = -4.9t^2 + 18t + 2$ where t is time in seconds and s is the height of the ball from the ground in meters.

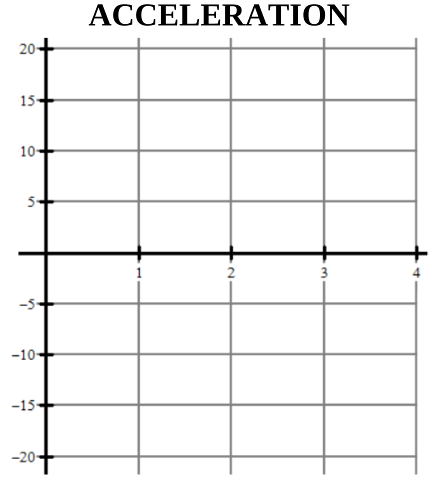
POSITION



VELOCITY



ACCELERATION



Particle Motion

The position (x-coordinate) of a particle moving on the x-axis is by $x(t) = t^3 - 4t^2 + 3$ for $t \geq 0$.

Find the displacement of the particle during the first 2 seconds.

Find the average velocity of the particle during the first 2 seconds.

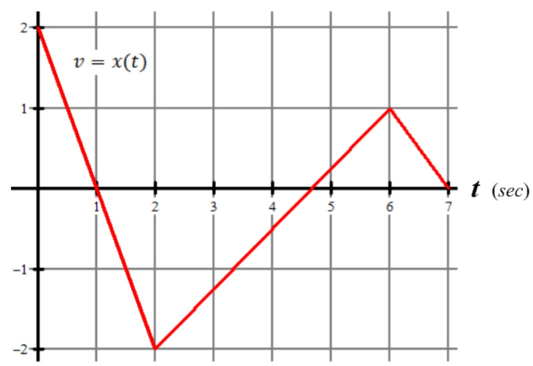
Find the instantaneous velocity of the particle when $t = 4$.

Find the acceleration of the particle when $t = 4$.

Describe the motion of the particle.

Particle Motion

The figure shows the velocity $v = x(t)$ of a particle moving on a coordinate line.



When does the particle move right? Move left? Speed up? Slow down?

When is the particle's acceleration Positive? Negative? Zero?

When does the particle have the greatest speed?

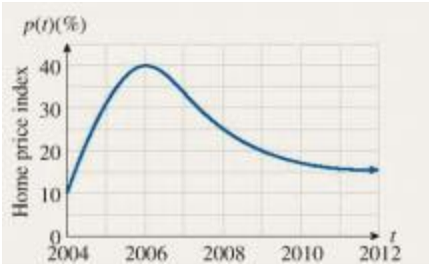
If velocity and acceleration have the same sign then object is

SUMMARY:

Now, summarize your notes here!

Use the information given to answer the following.

1. The home price index as a percentage change from 2003 in year *t*, is represented by $y = p(t)$.



a) What year does $p'(t) = 0$?

b) Is $p'(2008)$ positive, negative, or zero?

c) Find the average rate of change from 2006 to 2008.

2. A particle moves along a line so that its position at any time $t \geq 0$ is given by the function $s(t) = -t^3 + 7t^2 - 16t + 8$ where *s* is measured in meters and *t* is measured in seconds.

a) Find the instantaneous velocity at any time *t*.

b) Find the acceleration of the particle at any time *t*.

c) When is the particle at rest?

d) What is the displacement of the particle for the first 3 seconds?

3. A ball is drop off a 1200 foot cliff. The height of the ball over time is modeled by the function $h(t) = 1200 - 16t^2$ where *h* is height of the ball from the ground in feet and *t* is time in seconds.

a) Find $h'(3)$. Explain what it means.

b) Find $h''(3)$. Explain what it means.

4. The position, in meters, of a body at time *t* sec is $s(t) = t^3 - 6t^2 + 9t$. Find the body's acceleration each time the velocity is zero.

5. The following table shows oil production by Pemex, Mexico's national oil company, for 2001-2007 (*t* = 1 represents 2001)

***t* (year since 2000)**	1	3	5	7
***P* (million gallons/day)**	3.1	3.4	3.5	3.7

a) Approximate $P'(2)$. Label and justify!

6. A particle *P* moves on the number line. The graph $s = f(t)$ shows the position of *P* as function of time *t*.

a) When is *P* moving to the left?

b) When is *P* moving to the right?

c) When is *P* standing still?

d) Graph the particle's velocity where defined.

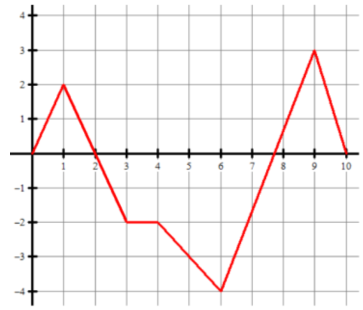
7. The number of iPods sold by Apple each year from 2004 through 2007 can be approximated by $f(t) = -t^2 + 20t + 3$ in millions of iPods where $t = 0$ represents 2004.

a) Is the number of iPods sold in 2006 increasing or decreasing?

b) What is the average rate of change from 2004-2007?

8. The figure shows the velocity $v = \frac{ds}{dt} = f(t)$ of a body moving along a coordinate line in meters per second.

a) When does the body reverse direction?



b) When is the body moving at a constant speed?

c) What is the body's maximum speed?

d) What time interval(s) is the body speeding up?

9. A rock thrown vertically upward from the surface of the moon at a velocity of 32 meters per second reaches a height of $s(t) = 32t - 0.8t^2$ meters in *t* seconds.

a) Find the rock's velocity and acceleration as functions of time.

b) How long did it take the rock to reach its highest point?

10. The table shows the cost *c* in dollars of a cup of coffee in *t* years after Starbucks opened its first store.

time, *t*	0	2	4	6	8
cost, *c*	1.20	1.35	1.45	1.75	2.00

a) Approximate $c'(5)$. Label and justify!

11. The data in the table gives selected values for the velocity, in meters per minute, of a particle moving along the *x*-axis. The velocity *v* is a differentiable function of time *t*.

Time *t*	0	2	5	6	8	12
Velocity $v(t)$	-3	2	3	5	7	5

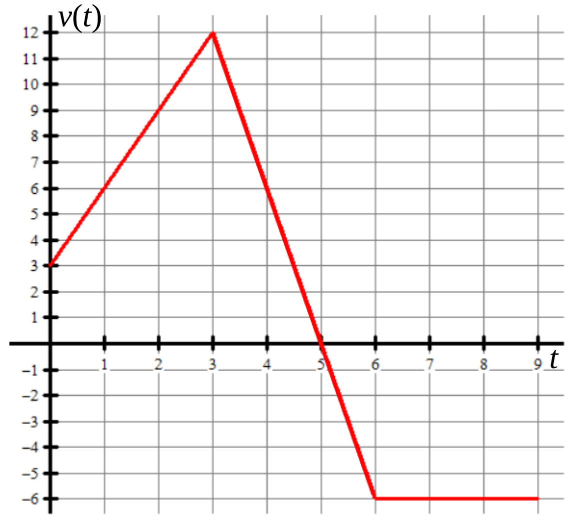
a) At $t = 0$, is the particle moving to the right or left? Explain.

b) Is there a time during the time interval $0 \le t \le 12$ minutes when the particle is at rest? Justify.

c) Use the data from the data to approximate $v'(10)$. Explain the meaning of $v'(10)$ in terms of the particle motion.

d) Let $a(t)$ denote the acceleration of the particle at time *t*. Is there guaranteed to be a time $t = c$ in the interval $0 \le t \le 12$ such that $a(c) = 0$? Justify.

12. The graph represents the velocity, in feet per second, of a particle moving along the x-axis over the time interval from $t = 0$ to $t = 9$ seconds.



a) At $t = 4$, is the particle moving to the right or left? Explain.

b) Over what time interval is the particle moving left? Explain.

c) At $t = 4$, is the acceleration positive or negative? Explain.

d) What is the average acceleration of the particle over the interval $2 \leq t \leq 4$? Show the computations and label your answer.

e) Is there guaranteed to be a time t in the interval $2 \leq t \leq 4$ such that $v'(t) = -\frac{3}{2}$ ft/sec^2 ? Justify.

f) At what time t in the given interval is the particle farthest to the right. Explain.

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13. A particle moves along the x-axis so that at time t its position is given by:

$$x(t) = t^3 - 6t^2 + 9t + 11$$ where t is measured in seconds and x is measured in feet

a) At $t = 0$, is the particle moving to the right or left? Explain.

b) At $t = 1$, is the velocity of the particle increasing or decreasing? Explain.

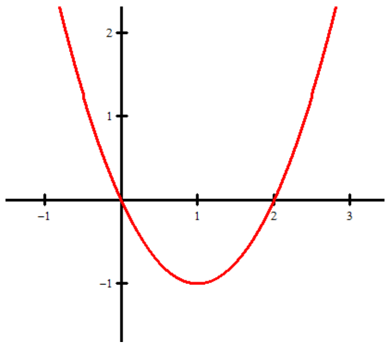
c) Find all values of t for which the particle is moving left.

d) What is the displacement of the first 6 seconds?

MULTIPLE CHOICE

1. The graph of the differentiable function $y = f(x)$ is shown below. Which of the following is true?

(A) $f'(0) > f(0)$
(B) $f'(1) < f(1)$
(C) $f'(2) < f(2)$
(D) $f'(1) = f(0)$
(E) $f'(2) = f(2)$



2. The position of the particle traveling along a straight line is $x(t) = t^3 - 9t^2 + 15t + 3$. On the interval $t = 0$ to $t = 10$, when is the particle farthest to the left?

(A) $t = 0$
(B) $t = 1$
(C) $t = 3$
(D) $t = 5$
(E) $t = 10$

3. If the position of an ant traveling along a horizontal path at time t is $3t^2 + 1$, what is the ant's average velocity from $t = 1$ to $t = 6$?

(A) $\frac{1}{21}$
(B) 6
(C) $\frac{109}{6}$
(D) 21
(E) 220

4. Find all values of c that satisfy the Mean Value Theorem for $f(x) = x^3 + 1$ on $[-2, 4]$.

(A) $c = 2$
(B) $c = \pm 2$
(C) $c = -2$
(D) $c = 0$
(E) No such value of c exists