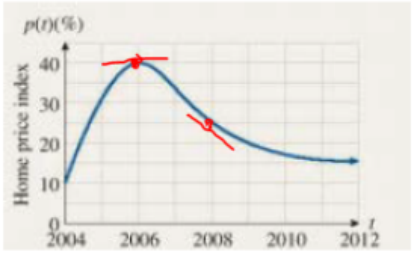
3.3 Velocity and other Rates of Change

PRACTICE

Use the information given to answer the following.

1. The home price index as a percentage change from 2003 in year t, is represented by $y = p(t)$.



a) What year does $p'(t) = 0$? 2006

b) Is $p'(2008)$ positive, negative, or zero?

c) Find the average rate of change from 2006 to 2008.

$\frac{40-25}{2006-2008} = -\frac{15}{2}$ percent change per year

2. A particle moves along a line so that its position at any time $t \geq 0$ is given by the function $s(t) = -t^3 + 7t^2 - 16t + 8$ where s is measured in meters and t is measured in seconds.

a) Find the instantaneous velocity at any time t.

$v(t) = -3t^2 + 14t - 16$

b) Find the acceleration of the particle at any time t.

$a(t) = -6t + 14$

c) When is the particle at rest?

$v'(t) = 0$ $t = 2$ and $\frac{8}{3}$ seconds

d) What is the displacement of the particle for the first 3 seconds?

-12 meters

3. A ball is drop off a 1200 foot cliff. The height of the ball over time is modeled by the function $h(t) = 1200 - 16t^2$ where h is height of the ball from the ground in feet and t is time in seconds.

$h'(t) = -32t$

a) Find $h'(3)$. Explain what it means.

At 3 seconds the ball is falling 96 feet per second

$h'(3) = -32(3) = -96$ ft per second

b) Find $h''(3)$. Explain what it means.

$h''(t) = -32$ At 3 seconds the ball's acceleration is -32 ft per sec^2

$h''(3) = -32$ feet per second^2

4. The position, in meters, of a body at time t sec is $s(t) = t^3 - 6t^2 + 9t$. Find the body's acceleration each time the velocity is zero.

6 and -6 meters per sec^2

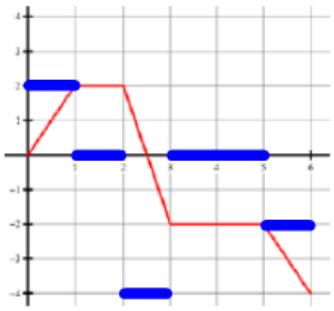
5. The following table shows oil production by Pemex, Mexico's national oil company, for 2001-2007 ($t = 1$ represents 2001)

t (year since 2000)	1	3	5	7
P (million gallons/day)	3.1	3.4	3.5	3.7

a) Approximate $P'(2)$. Label and justify!

$\frac{3.4-3.1}{3-1} = \frac{0.3}{2} = 0.15$ millions gallons per day per year

6. A particle P moves on the number line. The graph $s = f(t)$ shows the position of P as function of time t.



a) When is P moving to the left?

$(2,3)$ and $(5,6)$

b) When is P moving to the right?

$(0,1)$

c) When is P standing still?

$(1,2)$ and $(3,5)$

d) Graph the particle's velocity where defined.

graphed in blue above

7. The number of iPods sold by Apple each year from 2004 through 2007 can be approximated by $f(t) = -t^2 + 20t + 3$ in millions of iPods where $t = 0$ represents 2004.

a) Is the number of iPods sold in 2006 increasing or decreasing?

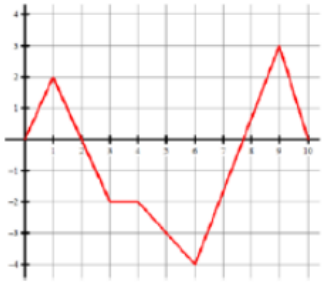
$f'(2) = -2(2) + 20 = 16$ positive slope

b) What is the average rate of change from 2004-2007?

$\frac{f(3)-f(0)}{3-0} = \frac{54-3}{3-0} = \frac{51}{3} = 17$ million ipods per year

8. The figure shows the velocity $v = \frac{ds}{dt} = f(t)$ of a body moving along a coordinate line in meters per second.

a) When does the body reverse direction? $t = 2$ $t \approx 7.8$



b) When is the body moving at a constant speed?

(3, 4)

c) What is the body's maximum speed?

4 meters per sec

d) What time interval(s) is the body speeding up?

(0, 1) and (2, 3) and (4, 6) and (≈7.8, 9)

9. A rock thrown vertically upward from the surface of the moon at a velocity of 32 meters per second reaches a height of $s(t) = 32t - 0.8t^2$ meters in t seconds.

a) Find the rock's velocity and acceleration as functions of time.

$s'(t) = v(t) = 32 - 1.6t$

$s''(t) = a(t) = -1.6$

b) How long did it take the rock to reach its highest point?

$v(t) = 0$

$32 - 1.6t = 0$

$-32 \quad -32$

$-1.6t = -32$

$\frac{-1.6t}{-1.6} = \frac{-32}{-1.6}$

$t = 20$ sec

10. The table shows the cost c in dollars of a cup of coffee in t years after Starbucks opened its first store.

time, t	0	2	4	6	8
cost, c	1.20	1.35	1.45	1.75	2.00

a) Approximate $c'(5)$. Label and justify!

$\frac{1.75-1.45}{6-4} = \frac{0.30}{2} = 0.15$ dollars per year

11. The data in the table gives selected values for the velocity, in meters per minute, of a particle moving along the x-axis. The velocity v is a differentiable function of time t.

Time t	0	2	5	6	8	12
Velocity $v(t)$	-3	2	3	5	7	5

a) At $t = 0$, is the particle moving to the right or left? Explain.

$v(0) = -3$ negative

b) Is there a time during the time interval $0 \le t \le 12$ minutes when the particle is at rest? Justify.

Yes, between $t = 0$ and $t = 2$ the velocity changes from negative to positive. Since the function is differentiable then it is also continuous then the Intermediate Value Theorem applies which states the particle must at some point be zero if changes from negative to positive.

c) Use the data from the data to approximate $v'(10)$. Explain the meaning of $v'(10)$ in terms of the particle motion.

$$\frac{v(12)-v(8)}{12-8} = \frac{5-7}{12-8} = \frac{-2}{4} = -\frac{1}{2} \text{ meters per min}^2$$

The particle is accelerating in the negative direction (left) at a rate of 0.5 meters per minute squared

d) Let $a(t)$ denote the acceleration of the particle at time t. Is there guaranteed to be a time $t = c$ in the interval $0 \le t \le 12$ such that $a(c) = 0$? Justify. Yes

Using the Mean Value Theorem since the function is differientiable. Since $\frac{v(12)-v(6)}{12-6} = 0$ There exists a point c such that $v'(c) = 0$

OR

Yes, between $t = 6$ and $t = 12$. The velocity is differientiable which means it is conituous which means we can use the Intermediate Value Theorem. The particle is increasing between $t = 6$ and $t = 8$ which means the acceleration is positive. The velocity is decreasing from $t = 8$ to $t = 12$ which the means the acceleration is negative. Since the acceleration changed from positive to negative there must be a time when the particle's acceleration was zero.

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12. The graph represents the velocity, in feet per second, of a particle moving along the x-axis over the time interval from $t = 0$ to $t = 9$ seconds.

a) At $t = 4$, is the particle moving to the right or left? Explain.

$v(4) = 6$ ft per sec positive

b) Over what time interval is the particle moving left? Explain.

$t = 5$ to $t = 9$ because the velocity is negative

c) At $t = 4$, is the acceleration positive or negative? Explain.

$v'(4) = a(4) = -6$ ft per sec^2

d) What is the average acceleration of the particle over the interval $2 \le t \le 4$? Show the computations and label your answer.

Acceleration is the rate of change of velocity so we find the average rate of change of the velocity.

$$\frac{v(4)-v(2)}{4-2} = \frac{6-9}{4-2} = -\frac{3}{2} \text{ ft per sec}^2$$

e) Is there guaranteed to be a time t in the interval $2 \le t \le 4$ such that $v'(t) = -\frac{3}{2}$ ft/sec^2 ? Justify.

No, the velocity is not differientiable at $t = 3$ so you cannot use the Intermediate Value Theorem

f) At what time t in the given interval is the particle farthest to the right. Explain.

At $t = 5$ the particle is farthest right. The particle is moving the positive direction from $t = 0$ to $t = 5$. The particle then moves left from $t = 5$ to $t = 9$ becuase the velocity is negative.

13. A particle moves along the x-axis so that at time t its position is given by:

$$x(t) = t^3 - 6t^2 + 9t + 11$$

a) At $t = 0$, is the particle moving to the right or left? Explain.

$x'(0) = 9$ positive

b) At $t = 1$, is the velocity of the particle increasing or decreasing? Explain.

$x''(1) = v'(1) = a(1) = -6$ negative

c) Find all values of t for which the particle is moving left.

$x'(t) = v(t) < 0$

$3t^2 - 12t + 9 < 0$

$3(t^2 - 4t + 3) < 0$

$3(t-1)(t-3) < 0$

$t = 1$ and 3

$1 < t < 3$

d) What is the displacement of the first 6 seconds?

$x(0) = 11$

$x(6) = 65$

54 feet

TEST PREP

1. **D**
2. **D**
3. **D**
4. **A**