Warm Up! Find the derivative of the following.

1. $y = \cos 2x$

$y' = -\sin(2x)\cdot 2$

$y' = -2\sin(2x)$

2. $f(x) = 2\sin x$

$f'(x) = 2\cos x$

3. $y = \cos^2 x$ $\quad [\cos x]^2$

$y' = 2[\cos x]^1 \cdot (-\sin x)$

$y' = -2\cos(x)\sin(x)$

4. $f(x) = \csc(\pi x)$

$f'(x) = -\csc(\pi x)\cot(\pi x)\pi$

$f'(x) = -\pi\csc(\pi x)\cot(\pi x)$

5. $y = -3\tan(5x^3)$

$y' = -3\sec^2(5x^3)(15x^2)$

$y' = -45x^2\sec^2(5x^3)$

6. $f(\theta) = 5\sec(4\theta)$

$f'(\theta) = 5\sec(4\theta)\tan(4\theta)\cdot 4$

$f'(\theta) = 20\sec(4\theta)\tan(4\theta)$

Warm Up! Evaluate the derivative at a point.

7. $f(x) = 3\sin(2x)$

$f'(x) = 3\cos(2x)\cdot 2$

$f'\left(\frac{\pi}{3}\right) = 3\cos\left(\frac{2\pi}{3}\right)\cdot 2$

$3\left(-\frac{1}{2}\right)\cdot 2$

$f'\left(\frac{\pi}{3}\right) = -3$

8. $f(\theta) = -2\csc\theta + 4$

$f'(\theta) = 2\csc\theta\cot\theta$

$f'\left(\frac{\pi}{2}\right) = 2\csc\left(\frac{\pi}{2}\right)\cot\left(\frac{\pi}{2}\right)$

$2(1)(0)$

$f'\left(\frac{\pi}{2}\right) = 0$

9. $y = 4\sin^3 x$ $\quad 4[\sin x]^3$

$12[\sin x]^2 \cdot \cos x$

$\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{4}}$

$12\left[\sin\left(\frac{\pi}{4}\right)\right]^2 \cdot \cos\left(\frac{\pi}{4}\right)$

$12\left[\frac{\sqrt{2}}{2}\right]^2\left(\frac{\sqrt{2}}{2}\right)$

$12\left(\frac{2}{4}\right)\left(\frac{\sqrt{2}}{2}\right)$

$\frac{6\sqrt{2}}{2} = 3\sqrt{2}$

Find the derivative of the following.

10. $f(x) = 2\sin x + \cos x$

$f'(x) = 2\cos(x) - \sin(x)$

11. $g(x) = 2x\cos(4x)$ $\quad$ Product Rule!

$u \quad v$

$u = 2x \quad v = \cos(4x)$

$u' = 2 \quad v' = -\sin(4x)\cdot 4$

$u'v + uv'$

$2\cos(4x) + 2x(-4\sin(4x))$

$g'(x) = 2\cos(4x) - 8x\cdot\sin(4x)$

12. $y = 5 - \csc\left(\frac{x^2}{2}\right)$

$y' = \csc\left(\frac{x^2}{2}\right)\cot\left(\frac{x^2}{2}\right)\cdot x$

13. $h(x) = \sqrt{\tan(2x)}$

$h(x) = \left[\tan(2x)\right]^{1/2}$

$h'(x) = \frac{1}{\cancel{2}}\left[\tan(2x)\right]^{-1/2}\sec^2(2x)\cdot\cancel{2}$

$h'(x) = \dfrac{\sec^2(2x)}{\sqrt{\tan(2x)}}$

14. $f(x) = \frac{1}{2}x - 2\sin^3(2x)$

$f'(x) = \frac{1}{2} - 12\sin^2(2x)\cos(2x)$

15. $y = \sec(\pi x + 1)$

$y' = \sec(\pi x + 1)\tan(\pi x + 1)\pi$

16. $r = \theta \sin\theta$ Product Rule!

$\quad\;\; u \quad v$

$r' = \sin\theta + \theta\cos\theta$

17. $s = t\cos(t^2)$ Product Rule!

$\quad\;\; u \quad v$

$u = t \qquad v = \cos(t^2) \qquad u'v + uv'$

$u' = 1 \qquad v' = -\sin(t^2)\cdot 2t$

$(1)\cos(t^2) + t\left(-2t\sin(t^2)\right)$

$s' = \cos(t^2) - 2t^2\sin(t^2)$

Evaluate the derivative at a point.

18. $f(x) = \cos(\tan x)$

$\cos(u)$

$\sin(u)\cdot u'$

$u = \tan x$

$f'(\pi) = 0$

$f'(x) = -\sin(\tan x)\sec^2 x$

$f'(\pi) = -\sin(\tan\pi)\sec^2\pi$

$f'(\pi) = -\sin(0)(-1)^2$

$f'(\pi) = -0(1)$

$f'(\pi) = 0$

19. $y = \frac{\sin x}{x}$ $\begin{matrix} u \\ v \end{matrix}$ Quotient Rule!

$u = \sin x \qquad v = x \qquad \dfrac{u'v - uv'}{v^2}$

$u' = \cos x \qquad v' = 1$

$\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{2}}$

$\dfrac{\cos(x)\cdot x - \sin(x)\cdot 1}{x^2}$

$\dfrac{\cos\left(\frac{\pi}{2}\right)\cdot\frac{\pi}{2} - \sin\left(\frac{\pi}{2}\right)}{\left(\frac{\pi}{2}\right)^2}$

$\dfrac{0\cdot\frac{\pi}{2} - 1}{\frac{\pi^2}{4}} = \dfrac{-1}{\frac{\pi^2}{4}} = -\dfrac{4}{\pi^2}$

Write the equation of the tangent line and the normal line at the point given.

20. $f(x) = \tan^2 x$ at $x = \frac{\pi}{4}$

Tangent Line

$y - 1 = 4\left(x - \frac{\pi}{4}\right)$

Normal Line

$y - 1 = -\frac{1}{4}\left(x - \frac{\pi}{4}\right)$

Particle Motion

21. The position of a particle moving along a coordinate line is $s(t) = 2\sin \pi t + 5\cos \pi t$, with s in meters and t in seconds. Find the particle's velocity and acceleration at $t = 1$.

position $s(t) = 2\sin(\pi t) + 5\cos(\pi t)$

velocity $s'(t) = 2\cos(\pi t)\cdot\pi - 5\sin(\pi t)\cdot\pi$

$s'(1) = 2\cos(\pi)\cdot\pi - 5\sin(\pi)\cdot\pi$

$s'(1) = 2(-1)\cdot\pi - 5(0)\cdot\pi$

$s'(1) = -2\pi$ meters per second

acceleration $s''(t) = -2\sin(\pi t)\cdot\pi^2 - 5\cos(\pi t)\cdot\pi^2$

$s''(1) = -2\sin(\pi)\cdot\pi^2 - 5\cos(\pi)\cdot\pi^2$

$s''(1) = -2(0)\cdot\pi^2 - 5(-1)\cdot\pi^2$

$s''(1) = 5\pi^2$ meters per second^2

MULTIPLE CHOICE

1. A
2. A
3. A
4. C
5. B

FREE RESPONSE

Your score: ____ out of 5

1. The rate of change, in kilometers per hour, of the altitude of a hot air balloon is given by $r(t) = t^3 - 4t^2 + 6$ for time $0 \leq t \leq 4$, where t is measured in hours. Assume the balloon is initially at ground level.

(a) For what values of t, $0 \leq t \leq 4$, is the altitude of the balloon decreasing?

$1.572 \leq t \leq 3.514$ hours ← 1 point

(b) Find the value of $r'(2)$ and explain the meaning of the answer in the context of the problem. Indicate units of measure.

$r'(2)$ = -4 km/hr per hour ← 1 point

At the second hour, the rate of change of the balloon's altitude is decreasing 4 km per hour^2.

↖ 1 point

(c) When does the hot air balloon have an acceleration of zero? Justify.

At $t = 2.\overline{6}$ hours ← 1 point

The derivative of r is the acceleration. $r'(2.\overline{6}) = 0$ ← 1 point