4.1 Exponential and Log Derivatives

Calculus

Name: Solutions

Practice

Find the derivative of each function.

1. $f(x) = e^{2x^2}$

$f'(x) = e^{2x^2} \cdot (4x)$

$\boxed{f'(x) = 4xe^{2x^2}}$

2. $f(x) = e^{x^4}$

$\boxed{f'(x) = 4x^3 e^{x^4}}$

3. $f(x) = \ln(2x^3)$

$f'(x) = \frac{1}{2x^3} \cdot 6x^2$

$f'(x) = \frac{6x^2}{2x^3}$

$\boxed{f'(x) = \frac{3}{x}}$

4. $f(x) = \ln(x - 5x^5)$

$\boxed{f'(x) = \frac{1-25x^4}{x-5x^5}}$

5. $f(x) = e^{\cos(7x^3)}$

$f'(x) = e^{\cos(7x^3)} \cdot (-\sin(7x^3)) \cdot 21x^2$

$\boxed{f'(x) = -21x^2 \sin(7x^3) e^{\cos(7x^3)}}$

6. $f(x) = e^{\sin(5x^9)}$

$\boxed{f'(x) = 45x^8 \cos(5x^9) e^{\sin(5x^9)}}$

7. $f(x) = \ln(x^6 + 5)$

$f'(x) = \frac{1}{x^6+5} \cdot (6x^5)$

$\boxed{f'(x) = \frac{6x^5}{x^6+5}}$

8. $f(x) = \ln\left(2x\sqrt{1+x}\right)$

$\boxed{f'(x) = \frac{1}{x} + \frac{1}{2+2x}}$

or

$\boxed{f'(x) = \frac{3x+2}{2x^2+2x}}$

9. $f(x) = 8^{\cos x}$

$f'(x) = 8^{\cos x} \ln(8) \cdot (-\sin x)$

$\boxed{f'(x) = -\sin x \ln(8)\, 8^{\cos x}}$

10. $f(x) = e^{x \sin x}$

$\boxed{f'(x) = (\sin x + x\cos x)e^{x \sin x}}$

11. $f(x) = \log_7(x^4)$

$f'(x) = \frac{1}{x^4} \cdot \frac{1}{\ln 7} \cdot 4x^3$

$\boxed{f'(x) = \frac{4}{x \ln 7}}$

12. $f(x) = \ln x \log x$

$\boxed{f'(x) = \frac{2\log x}{x}}$

Hint: $\frac{\ln x}{\ln 10} = \log x$

13. $f(x) = \ln(\sin 4x) - x^4$

$f'(x) = \frac{1}{\sin(4x)} \cdot \cos(4x) \cdot 4 - 4x^3$

$\boxed{f'(x) = 4\cot(4x) - 4x^3}$

14. $f(x) = e^{\pi x} - \ln(e^{\pi x})$

$\boxed{f'(x) = \pi e^{\pi x} - \pi}$

15. $f(x) = e^{-5x} \cos 2x$

$\boxed{f'(x) = -\frac{5\cos 2x + 2\sin 2x}{e^{5x}}}$

16. $f(x) = \frac{e^{\tan 3x}}{3}$

$f(x) = \frac{1}{3} e^{\tan 3x}$

$f'(x) = \frac{1}{3} e^{\tan 3x} \cdot \sec^2 3x \cdot 3$

$\boxed{f'(x) = e^{\tan 3x} \cdot \sec^2 3x}$

17. $f(x) = 2^{\tan x}$

$$\boxed{f'(x) = 2^{\tan x}(\ln 2)\sec^2 x}$$

18. $f(x) = \log\sqrt{10^{5x}}$

If you use properties of logs first, this problem is much easier!

$\log_{10} 10^{\frac{5x}{2}}$

$\frac{5x}{2}$

$$\boxed{f'(x) = 5/2}$$

19. $f(x) = \frac{x}{e^{3x}}$

$f'(x) = \frac{(1)e^{3x} - xe^{3x}\cdot 3}{e^{6x}}$

$f'(x) = \frac{e^{3x} - 3xe^{3x}}{e^{6x}}$

$f'(x) = \frac{e^{3x}(1-3x)}{e^{6x}}$

$$\boxed{f'(x) = \frac{1-3x}{e^{3x}}}$$

20. $f(x) = x^7 7^x$

$$\boxed{f'(x) = 7x^6 7^x + x^7 7^x \ln 7}$$

21. $f(x) = \ln 11^x$

Apply properties of logarithms first!

$f(x) = x\ln 11$

Now take the derivative.

$$\boxed{f'(x) = \ln 11}$$

22. $f(x) = e^{2x} - 2^{ex}$

$$\boxed{f'(x) = 2e^{2x} - e\ln(2)\,2^{ex}}$$

23. $f(x) = \cos(\ln(2x^2))$

$f'(x) = -\sin(\ln(2x^2))\cdot\frac{1}{2x^2}\cdot 4x$

$$\boxed{f'(x) = -\frac{2}{x}\sin(\ln(2x^2))}$$

24. If $f(x) = e^{x^2}$, what is the equation of the tangent line at $x = 1$.

$y - e = 2e(x-1)$

or

$y = 2ex - e$

25. At what coordinate point(s) is the tangent line of $f(x) = \ln(x^3)$ parallel to $y = 7 + 2x$.

Slope

$2 = \frac{3}{x}$

$2x = 3$

$x = 3/2$

$f'(x) = \frac{1}{x^3}\cdot 3x^2$

$f'(x) = \frac{3}{x}$

$f(\frac{3}{2}) = \ln(\frac{27}{8})$

$$\boxed{\left(\frac{3}{2}, \ln\left(\frac{27}{8}\right)\right)}$$

26. $f(x) = \ln(x^2)$ on the interval $1 < x < e$. On this interval, when will the average rate of change equal the instantaneous rate of change. [This is applying the Mean Value Theorem]

$$\boxed{\text{At } x = e-1}$$

27. Find the values of x where the tangent to the graph of $y = e^{2x}$ is parallel to $12x - 2y = 6$

$y' = 2e^{2x}$

$-2y = -12x + 6$

$y = 6x - 3$

Slope

$2e^{2x} = 6$

$e^{2x} = 3$

$\ln(e^{2x}) = \ln 3$

$2x = \ln 3$

$\boxed{x = \frac{\ln 3}{2}}$

28. Find the values of x where the tangent to the graph of $y = \frac{1}{e^{3x}}$ is parallel to $5x + y = 109$

$\boxed{x = \frac{\ln \frac{5}{3}}{-3}}$

Test Prep: 1A, 2A, 3C, 4B, 5C

<u>Free Response Scoring Guide</u>

Use this only AFTER you have attempted the problem on your own.

<u>Solutions</u>	<u>Points</u>
(a) $a(4) = v'(4) = \frac{5}{7}$	1 : answer
(b) $v(t) = 0$ $t^2 - 3t + 3 = 1$ $t^2 - 3t + 2 = 0$ $(t-2)(t-1) = 0$ $t = 1, 2$ $v(t) > 0$ for $0 < t < 1$ $v(t) < 0$ for $1 < t < 2$ $v(t) > 0$ for $2 < t < 5$ The particle changes direction when $t = 1$ and $t = 2$. The particle travels to the left when $1 < t < 2$.	$3 : \begin{cases} 1 : \text{sets } v(t) = 0 \\ 1 : \text{direction change at } t = 1, 2 \\ 1 : \text{interval with reason} \end{cases}$