Calculus

Write your questions and thoughts here!

4.2 Inverse Derivatives

Name:_____

Notes

Recall: Given $f(x) = 5(x-2)^3 - 1$, find $f^{-1}(x)$.

CONFUSING NOTATION:	Reciprocal		Inverse
	$x^{-1} =$	or	$f^{-1}(x)$ **means**

Quote from the AP Exam:
"Notation: The inverse of a trigonometric function x may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$)."

Inverse Trig Derivatives:

$\frac{d}{dx}\sin^{-1}(x) =$	$\frac{d}{dx}\cos^{-1}(x) =$
$\frac{d}{dx}\sec^{-1}(x) =$	$\frac{d}{dx}\csc^{-1}(x) =$
$\frac{d}{dx}\tan^{-1}(x) =$	$\frac{d}{dx}\cot^{-1}(x) =$

Find the derivative.

1. $\frac{d}{dx}\sin^{-1}(3x)$

2. $\frac{d}{dx}\tan^{-1}(2x^2)$

3. $\frac{d}{dx}\sec^{-1}(5x^6)$

4. What is the anti-derivative of $\frac{12x}{\sqrt{1-36x^4}}$?

Derivative of an Inverse Function:

$$\frac{d}{dx}[f^{-1}(x)] =$$

The functions f and g are differentiable. For all x, $f(g(x)) = x$ and $g(f(x)) = x$.

5. If $f(12) = 4$ and $f'(12) = -5$, find $g(4)$ and $g'(4)$.

6. If $f(3) = -2$ and $f'(3) = 5$, find $g(-2)$ and $g'(-2)$.

4.2 Inverse Derivatives

Calculus Name: _____ **Practice**

Find the following.

1. $\frac{d}{dx}\sin^{-1}(5x)$	2. $\frac{d}{dx}\csc^{-1}(4x^5)$	3. $\frac{d}{dx}\tan^{-1}(2x)$
4. $\frac{d}{dx}\frac{\sin x}{x}$	5. $\frac{d}{dx}\sec^{-1}(x^3)$	6. $\frac{d}{dx}\csc 6x$
7. $\lim_{x\to 2}\frac{x-2}{x^2+5x-14}$	8. $\frac{d}{dx}\cos^{-1}(3x^2)$	9. Anti-derivative of $f'(x)=\frac{5}{\sqrt{1-25x^2}}$
10. $\frac{d}{dx}\cot^{-1}(-x)$	11. Anti-derivative of $f'(x)=-\frac{6x^2}{1+4x^6}$	12. $\frac{d}{dx}\log_5 4x$
13. $\frac{d}{dx}\cos^{-1}(-7x)$	14. $\frac{d}{dx}\csc^{-1}(x^7)$	15. $\frac{d}{dx}\cot^{-1}(4x^4)$
16. $\frac{d}{dx}e^{2x^5}$	17. $\frac{d}{dx}\tan^{-1}(\sqrt{x})$	18. $\frac{d}{dx}5x\sin^{-1}(2x^2)$

19. Anti-derivative of $f'(x) = \frac{7}{|x|\sqrt{9x^{14}-1}}$

20. $\frac{d}{dx}\tan(e^x)$

21. $\frac{d}{dx}\sec^{-1}(3\ln x)$

22. $\frac{d}{dx}\sin^{-1}(2x)$

23. $\frac{d}{dx}\frac{15x^3+3x^2+55x}{3x}$

24. Anti-derivative of $f'(x) = -\frac{8x}{\sqrt{1-16x^4}}$

25. What is the equation of the line tangent to the curve $y = \arcsin(x)$ at the point where $x = \frac{\sqrt{2}}{2}$?

26. What is the equation of the line tangent to the curve $y = \arccos(4x)$ at the point where $x = \frac{\sqrt{3}}{8}$?

The functions f and g are differentiable. For all x, $f(g(x)) = x$ and $g(f(x)) = x$.

27. If $f(1) = 5$ and $f'(1) = -2$, find $g(5)$ and $g'(5)$.

28. If $f(-3) = 7$ and $f'(-3) = 8$, find $g(7)$ and $g'(7)$.

29. If $f(2) = -3$ and $f'(2) = 11$, find $g(-3)$ and $g'(-3)$.

30. If $f(8) = 1$ and $f'(8) = 6$, find $g(1)$ and $g'(1)$.

31. If $f(-1) = 6$ and $f'(-1) = -3$, find $g(6)$ and $g'(6)$.

32. If $f(-8) = -1$ and $f'(-8) = 7$, find $g(-1)$ and $g'(-1)$.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	5	-5	4	5
2	1	-6	3	3
3	6	4	1	6
4	2	9	6	1
5	3	1	1	2
6	4	2	2	4

f and g are differentiable functions. Using the table above, find the following. f and g are NOT inverses!

33. $g^{-1}(4)$

34. $f^{-1}(5)$

35. $\frac{d}{dx}g^{-1}(3)$

36. $\frac{d}{dx}f^{-1}(1)$

37. Find the line tangent to the graph of $f^{-1}(x)$ at $x = 1$.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
5	6	-2	6	-5
6	9	2	5	-4
7	8	-10	10	9
8	10	4	9	2
9	5	5	7	10
10	7	7	8	6

f and g are differentiable functions. Using the table above, find the following. f and g are NOT inverses!

38. $g^{-1}(7)$

39. $f^{-1}(7)$

40. $\frac{d}{dx}f^{-1}(6)$

41. $\frac{d}{dx}g^{-1}(8)$

42. Find the line tangent to the graph of $g^{-1}(x)$ at $x = 8$.

4.2 Inverse Derivatives

Test Prep

1. Compute the derivative of $f(x) = \ln x - \sin x + \arctan x + 2^x, x > 0$.

(A) $f(x) = \frac{1}{x} - \cos x + \frac{1}{1+x^2} + x2^x$

(B) $f(x) = \frac{1}{x} - \cos x + \frac{1}{1-x^2} + x2^x$

(C) $f(x) = \frac{1}{x} + \cos x + \frac{1}{1-x^2} + (\ln 2)2^x$

(D) $f(x) = \frac{1}{x} - \cos x + \frac{1}{1+x^2} + (\ln 2)2^x$

(E) $f(x) = \frac{1}{x} + \cos x + \frac{1}{1+x^2} + (\ln 2)2^x$

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2. What is an equation for the line tangent to $y = \tan^{-1} x$ at $x = \sqrt{3}$?

(A) $y - \frac{\pi}{3} = -\frac{1}{2}(x - \sqrt{3})$

(B) $y - \frac{\pi}{6} = -\frac{1}{4}(x - \sqrt{3})$

(C) $y - \frac{\pi}{3} = -\frac{1}{4}(x - \sqrt{3})$

(D) $y - \frac{\pi}{6} = \frac{3}{4}(x - \sqrt{3})$

(E) $y - \frac{\pi}{3} = \frac{1}{4}(x - \sqrt{3})$

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3. $\lim_{x\to-\infty} \frac{2x+3}{\sqrt{x^2+x+1}}$ is

(A) -2 (B) -1 (C) 0 (D) 2 (E) nonexistent

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4. If $f(x) = 3x^2 - x$, and $g(x) = f^{-1}(x)$, then $g'(10)$ could be

(A) 59 (B) $\frac{1}{59}$ (C) $\frac{1}{10}$ (D) 11 (E) $\frac{1}{11}$

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5. If $f(x) = x^{\frac{5}{2}}$, then $f'(4)$?

(A) -10 (B) 24 (C) 5 (D) 10 (E) 20