4.2 Inverse Derivatives Calculus

Name: Solutions

Practice

Find the following.

1. $\frac{d}{dx}\sin^{-1}(5x)$

$\frac{1}{\sqrt{1-(5x)^2}}\cdot 5$

$\boxed{\frac{5}{\sqrt{1-25x^2}}}$

2. $\frac{d}{dx}\csc^{-1}(4x^5)$

$\boxed{-\frac{5}{|x|\sqrt{16x^{10}-1}}}$

3. $\frac{d}{dx}\tan^{-1}(2x)$

$\frac{1}{(2x)^2+1}\cdot 2$

$\boxed{\frac{2}{4x^2+1}}$

4. $\frac{d}{dx}\frac{\sin x}{x}$

$\boxed{\frac{x\cos x-\sin x}{x^2}}$

5. $\frac{d}{dx}\sec^{-1}(x^3)$

$\frac{1}{|x^3|\sqrt{(x^3)^2-1}}\cdot 3x^2$ $\frac{\text{Pos.}}{\text{Pos.}}$

$\boxed{\frac{3}{|x|\sqrt{x^6-1}}}$

6. $\frac{d}{dx}\csc 6x$

$\boxed{-6\csc(6x)\cot(6x)}$

7. $\lim\limits_{x\to 2}\frac{x-2}{x^2+5x-14}$

$\frac{x-2}{(x-2)(x+7)}$

$\lim\limits_{x\to 2}\frac{1}{x+7}$

$\frac{1}{2+7}=\boxed{1/9}$

8. $\frac{d}{dx}\cos^{-1}(3x^2)$

$\boxed{-\frac{6x}{\sqrt{1-9x^4}}}$

9. Anti-derivative of $f'(x)=\frac{5}{\sqrt{1-25x^2}}$

$\boxed{\sin^{-1}(5x)+C}$

10. $\frac{d}{dx}\cot^{-1}(-x)$

$\boxed{\frac{1}{x^2+1}}$

11. Anti-derivative of $f'(x)=-\frac{6x^2}{1+4x^6}$

$\boxed{\cot^{-1}(2x^3)+C}$

12. $\frac{d}{dx}\log_5 4x$

$\boxed{\frac{1}{x\ln 5}}$

13. $\frac{d}{dx}\cos^{-1}(-7x)$

$-\frac{1}{\sqrt{1-(-7x)^2}}\cdot(-7)$

$\boxed{\frac{7}{\sqrt{1-49x^2}}}$

14. $\frac{d}{dx}\csc^{-1}(x^7)$

$\boxed{-\frac{7}{|x|\sqrt{x^{14}-1}}}$

15. $\frac{d}{dx}\cot^{-1}(4x^4)$

$-\frac{1}{(4x^4)^2+1}\cdot 16x^3$

$\boxed{-\frac{16x^3}{16x^8+1}}$

16. $\frac{d}{dx}e^{2x^5}$

$\boxed{10x^4e^{2x^5}}$

17. $\frac{d}{dx}\tan^{-1}(\sqrt{x})$

$\frac{1}{(\sqrt{x})^2+1}\cdot\frac{1}{2\sqrt{x}}$

$\boxed{\frac{1}{2\sqrt{x}(x+1)}}$

18. $\frac{d}{dx}5x\sin^{-1}(2x^2)$

$\boxed{5\sin^{-1}(2x^2)+\frac{20x^2}{\sqrt{1-4x^4}}}$

19. Anti-derivative of $f'(x) = \frac{7}{|x|\sqrt{9x^{14}-1}}$

$\sqrt{9x^{14}} = 3x^7$

$\boxed{\sec^{-1}(3x^7) + C}$

20. $\frac{d}{dx}\tan(e^x)$

$\boxed{e^x \sec^2(e^x)}$

21. $\frac{d}{dx}\sec^{-1}(3\ln x)$

$\frac{1}{3|\ln x|\sqrt{9(\ln x)^2-1}} \cdot \frac{3}{x}$

$\boxed{\frac{1}{x|\ln x|\sqrt{9(\ln x)^2-1}}}$

22. $\frac{d}{dx}\sin^{-1}(2x)$

$\boxed{\frac{2}{\sqrt{1-4x^2}}}$

23. $\frac{d}{dx}\frac{15x^3+3x^2+55x}{3x}$

$\frac{d}{dx}\left(\frac{15x^3}{3x}+\frac{3x^2}{3x}+\frac{55x}{3x}\right)$

$\frac{d}{dx}\left(5x^2+x+\frac{55}{3}\right)$

$\boxed{10x+1}$

24. Anti-derivative of $f'(x) = -\frac{8x}{\sqrt{1-16x^4}}$

$\boxed{\cos^{-1}(4x^2) + C}$

25. What is the equation of the line tangent to the curve $y = \arcsin(x)$ at the point where $x = \frac{\sqrt{2}}{2}$?

$y = \arcsin\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

$y'\left(\frac{\sqrt{2}}{2}\right) = \frac{1}{\sqrt{1-\left(\frac{2}{4}\right)}} = \frac{1}{\sqrt{\frac{1}{2}}} = \frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}$

$\boxed{y - \frac{\pi}{4} = \sqrt{2}\left(x - \frac{\sqrt{2}}{2}\right)}$

26. What is the equation of the line tangent to the curve $y = \arccos(4x)$ at the point where $x = \frac{\sqrt{3}}{8}$?

$y = \arccos\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

$y' = -\frac{4}{\sqrt{1-16x^2}}$

$y' = -\frac{4}{\sqrt{1-16\left(\frac{3}{64}\right)}} = -\frac{4}{\sqrt{\frac{1}{4}}} = -\frac{4}{\frac{1}{2}} = -8$

$\boxed{y - \frac{\pi}{6} = -8\left(x - \frac{\sqrt{3}}{8}\right)}$

The functions f and g are differentiable. For all x, $f(g(x)) = x$ and $g(f(x)) = x$.

27. If $f(1) = 5$ and $f'(1) = -2$, find $g(5)$ and $g'(5)$.

$g'(5) = \frac{1}{f'(g(5))} = \frac{1}{f'(1)}$

$\boxed{g(5) = 1 \qquad g'(5) = -\frac{1}{2}}$

28. If $f(-3) = 7$ and $f'(-3) = 8$, find $g(7)$ and $g'(7)$.

$g'(7) = \frac{1}{f'(g(7))} = \frac{1}{f'(-3)}$

$\boxed{g(7) = -3 \qquad g'(7) = \frac{1}{8}}$

29. If $f(2) = -3$ and $f'(2) = 11$, find $g(-3)$ and $g'(-3)$.

$g'(-3) = \frac{1}{f'(g(-3))} = \frac{1}{f'(2)}$

$\boxed{g(-3) = 2 \qquad g'(-3) = \frac{1}{11}}$

30. If $f(8) = 1$ and $f'(8) = 6$, find $g(1)$ and $g'(1)$.

$g'(1) = \frac{1}{f'(g(1))} = \frac{1}{f'(8)}$

$\boxed{g(1) = 8 \qquad g'(1) = \frac{1}{6}}$

31. If $f(-1) = 6$ and $f'(-1) = -3$, find $g(6)$ and $g'(6)$.

$g'(6) = \frac{1}{f'(g(6))} = \frac{1}{f'(-1)}$

$\boxed{g(6) = -1 \quad g'(6) = -\frac{1}{3}}$

32. If $f(-8) = -1$ and $f'(-8) = 7$, find $g(-1)$ and $g'(-1)$.

$g'(-1) = \frac{1}{f'(g(-1))} = \frac{1}{f'(-8)}$

$\boxed{g(-1) = -8 \quad g'(-1) = \frac{1}{7}}$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	5	−5	4	5
2	1	−6	3	3
3	6	4	1	6
4	2	9	6	1
5	3	1	1	2
6	4	2	2	4

***f* and *g* are differentiable functions. Using the table above, find the following.**

33. $g^{-1}(4)$

$\boxed{1}$

34. $f^{-1}(5)$

$\boxed{1}$

35. $\frac{d}{dx} g^{-1}(3)$

$\frac{1}{g'(g^{-1}(3))} = \frac{1}{g'(2)} = \boxed{\frac{1}{3}}$

36. $\frac{d}{dx} f^{-1}(1)$

$\frac{1}{f'(f^{-1}(1))} = \frac{1}{f'(2)} = \boxed{-\frac{1}{6}}$

37. Find the line tangent to the graph of $f^{-1}(x)$ at $x = 1$.

$\boxed{y - 2 = -\frac{1}{6}(x - 1)}$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
5	6	−2	6	−5
6	9	2	5	−4
7	8	−10	10	9
8	10	4	9	2
9	5	5	7	10
10	7	7	8	6

***f* and *g* are differentiable functions. Using the table above, find the following. *f* and *g* are NOT inverses!**

38. $g^{-1}(7)$

$\boxed{9}$

39. $f^{-1}(7)$

$\boxed{10}$

40. $\frac{d}{dx} f^{-1}(6)$

$\frac{1}{f'(f^{-1}(6))} = \frac{1}{f'(5)} = \boxed{-\frac{1}{2}}$

41. $\frac{d}{dx} g^{-1}(8)$

$\frac{1}{g'(g^{-1}(8))} = \frac{1}{g'(10)} = \boxed{\frac{1}{6}}$

42. Find the line tangent to the graph of $g^{-1}(x)$ at $x = 8$.

$\boxed{y - 10 = \frac{1}{6}(x - 8)}$

Test Prep: 1D, 2E, 3A, 4E, 5E