Calculus

Write your questions and thoughts here!

4.3 L'Hôpital's Rule

Name:________________________

Notes

Recall: Special Trig Limits $\lim_{x\to 0}\frac{\sin x}{x}=1$ and $\lim_{x\to 0}\frac{1-\cos x}{x}=0$

Recall: When evaluating limits, first try direct substitution! $\lim_{x\to 3}\frac{2x-5}{x}=$

Example 1: $\lim_{x\to 2}\frac{x^2-7x+10}{x-2}=$

L'Hôpital's Rule:

Suppose $f(a)=0$ and $g(a)=0$ and $\lim_{x\to 0}\frac{f(x)}{g(x)}=\frac{0}{0}$ or $\frac{\infty}{\infty}$. L'Hopital's Rule allows you to apply the following:

Evaluate each limit. Use L'Hôpital's when possible.

2. $\lim_{x\to 2}\frac{x-2}{3x^3-6x^2+x-2}$	3. $\lim_{x\to 0}\frac{\sin(6x)}{x}$	4. $\lim_{x\to 0}\frac{1-\cos(x)}{x^2}$

L'HÔPITAL'S IS NOT THE QUOTIENT RULE!!

5. $\frac{d}{dx}\frac{\sin(6x)}{x}$

Now summarize what you learned!

4.3 L'Hôpital's Rule

Calculus Name: ______________________________ **Practice**

Find the following. Use L'Hôpital's when possible.

1. $\lim_{x\to 1}\frac{x-1}{x^2-3x+2}$	2. $\lim_{x\to -5}\frac{x^2-2x-35}{x+5}$	3. $\lim_{x\to 0}\frac{4x}{\ln(x+1)}$
4. $\lim_{x\to 0}\frac{x-1}{x^2-3x+2}$	5. $\lim_{x\to 1}\frac{2(x^2-1)}{\ln x^2}$	6. $\frac{d}{dx}\frac{6x^2+x}{\sin(x)}$
7. $\lim_{x\to 0}\frac{2x^2}{e^x-1-x}$	8. $\lim_{x\to 0}\frac{2x^2}{1-\cos(4x)}$	9. $\lim_{x\to 0}\frac{\sqrt{4+x}-2}{x}$
10. $\lim_{x\to -3}\frac{x-1}{x^2+7x+10}$	11. $\frac{d}{dx}\frac{6x^2+x}{x+1}$	12. $\lim_{x\to 0}\frac{cosx-1}{x^2}$
13. $\lim_{x\to\infty}\frac{e^{2x}}{2x^2}$	14. $\lim_{x\to\infty}\frac{\ln x^2}{\ln(x+4)^3}$	15. $\lim_{x\to -2}\frac{x+2}{x^2+2x-3}$

16. $\frac{d}{dx}\frac{e^x}{\cos(2x)}$

17. $\lim_{x\to 0}\frac{\sqrt{1+x}-1-\frac{x}{2}}{x^2}$

18. $\lim_{x\to 10}\frac{5-\sqrt{x+15}}{x-10}$

19. $\lim_{x\to -5}\frac{x^2-2x-15}{x+5}$

4.3 L'Hôpital's Rule

Test Prep

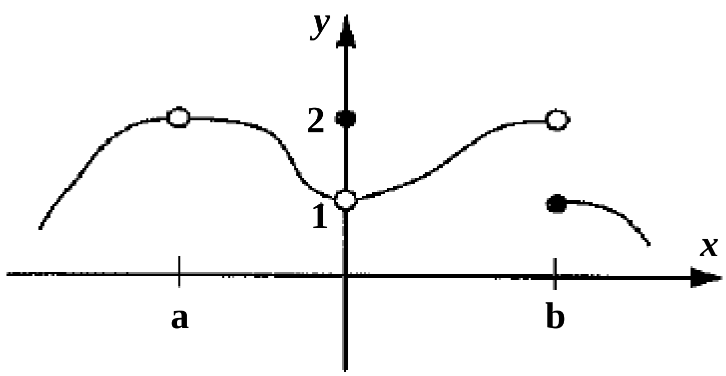
1. $\lim_{x\to 2}\frac{e^{2x}-e^4}{x-2} =$

(A) e (B) $2e$ (C) $2e^2$ (D) e^4 (E) $2e^4$

2. If $f(x) = x\sqrt{4x-1}$, then $f'(x)$ is

(A) $\frac{6x-1}{\sqrt{4x-1}}$ (B) $\frac{2x}{\sqrt{4x-1}}$ (C) $\frac{1}{\sqrt{4x-1}}$

(D) $\frac{-6x+1}{\sqrt{4x-1}}$ (E) $\frac{9x-2}{2\sqrt{4x-1}}$



3. The graph of the function f is shown in the figure above. Which of the following statements about f is true?

(A) $f(a)$ exists

(B) $\lim_{x \to a} f(x) = 2$

(C) $\lim_{x \to b} f(x) = 1$

(D) $\lim_{x \to b^-} f(x) = \lim_{x \to b^+} f(x)$

(E) f is continuous at $x = 0$

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4. Let f be a function defined for all real numbers. Which of the following statements about f must be true?

(A) If $\lim_{x \to 2} f(x) = 7$, then $f(2) = 7$.

(B) If $\lim_{x \to 5} f(x) = -3$, then -3 is in the range of f.

(C) If $\lim_{x \to 3^-} f(x) \neq \lim_{x \to 3^+} f(x)$, then $\lim_{x \to 3} f(x)$ does not exist.

(D) If $\lim_{x \to 4} f(x)$ does not exist, then $f(4)$ does not exist.

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5. The slope to the tangent line to the graph of $y = \tan(2x)$ at $x = \frac{\pi}{8}$ is

(A) $\frac{1}{\sqrt{2}}$

(B) $\sqrt{2}$

(C) 2

(D) $2\sqrt{2}$

(E) 4

6. An object moves along the y-axis with the coordinate position $y(t)$ and velocity $v(t) = \sqrt{t} - \cos(e^t)$ for $t \geq 0$. At time $t = 1$, the object is

(A) moving downward with negative acceleration
(B) moving upward with negative acceleration
(C) moving downward with positive acceleration
(D) moving upward with positive acceleration
(E) at rest

FREE RESPONSE **Your score: _______ out of 4**

Use the space below the problem to show work and solutions. Score your answers when completed.

1. A 12,000-liter tank of water is filled to capacity. At time $t = 0$, water begins to drain out of the tank at a rate modeled by $r(t)$, measured in liters per hour, where r is given by the piecewise-defined function

$$r(t) = \begin{cases} \dfrac{600t}{t+3} & \text{for } 0 \leq t \leq 5 \\ 1000e^{-0.2t} & \text{for } t > 5 \end{cases}$$

(a) Is r continuous at $t = 5$? Show the work that leads to your answer.

(b) Find $r'(3)$. Using correct units, explain the meaning of that value in the context of this problem.