Name: Solutions Date:______________ Period: _____ **Review**

4 Review – More Derivatives

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you must study all packets from Unit 4.

4.1 Exponential and Logarithmic Derivatives:

Exponential Derivatives:

$$\frac{d}{dx}e^u = e^u \cdot u'$$

$$\frac{d}{dx}a^u = a^u \cdot u' \cdot \ln a$$

Logarithmic Derivatives:

$$\frac{d}{dx}\ln u = \frac{1}{u} \cdot u'$$

$$\frac{d}{dx}\log_a u = \frac{1}{u} \cdot \frac{1}{\ln a} \cdot u'$$

Find the derivative of each function.

1. $f(x) = e^{4x^3}$ $\boxed{f'(x) = 12x^2 e^{4x^3}}$	2. $f(x) = 5\ln(8x)$ $\boxed{f'(x) = \frac{5}{x}}$
3. $f(x) = e^{\sin(x^4)}$ $\boxed{f'(x) = 4x^3\cos(x^4)e^{\sin(x^4)}}$	4. $f(x) = \ln(\cos 5x)$ $f'(x) = \frac{-5\sin(5x)}{\cos(5x)} = \boxed{-5\tan(5x)}$
5. $f(x) = \log_4(x^2)$ $\boxed{f'(x) = \frac{2}{\ln(4)\, x}}$	6. $f(x) = 3^{\tan x}$ $\boxed{f'(x) = \ln(3)\sec^2 x \cdot 3^{\tan x}}$

4.2 Inverse Derivatives:

Inverse Trig Derivatives:

$$\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\cos^{-1}(x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\sec^{-1}(x) = \frac{1}{|x|\sqrt{x^2-1}} \qquad \frac{d}{dx}\csc^{-1}(x) = -\frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}\tan^{-1}(x) = \frac{1}{x^2+1} \qquad \frac{d}{dx}\cot^{-1}(x) = -\frac{1}{x^2+1}$$

Derivative of an Inverse Function:

$$\frac{d}{dx}[f^{-1}(x)] = \frac{1}{f'[f^{-1}(x)]}$$

7. $\frac{d}{dx}\sin^{-1}(8x)$

$\boxed{\frac{8}{\sqrt{1-64x^2}}}$

8. $\frac{d}{dx}\sec^{-1}(x^2)$

$\boxed{\frac{2}{x\sqrt{x^4-1}}}$

9. $\frac{d}{dx}\cot^{-1}(2x)$

$\boxed{-\frac{2}{4x^2+1}}$

The functions f and g are differentiable. For all x, $f(g(x)) = x$ and $g(f(x)) = x$.

10. If $f(6) = -1$ and $f'(6) = 3$, find $g(-1)$ and $g'(-1)$.

$g'(-1) = \frac{1}{f'(g(-1))} = \frac{1}{f'(6)}$

$\boxed{g(-1) = 6}$ $\boxed{g'(-1) = \frac{1}{3}}$

11. If $g(-8) = 7$ and $g'(-8) = -5$, find $f(7)$ and $f'(7)$.

$f'(7) = \frac{1}{g'(f(7))} = \frac{1}{g'(8)}$

$\boxed{f(7) = -8}$ $\boxed{f'(7) = -\frac{1}{5}}$

4.3 L'Hopital's Rule:

L'Hôpital's Rule:

Suppose $f(a) = 0$ and $g(a) = 0$ and $\lim_{x\to 0}\frac{f(x)}{g(x)} = \frac{0}{0}$ or $\frac{\infty}{\infty}$.

L'Hopital's Rule allows you to apply the following:

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{f'(a)}{g'(a)}$$

L'HÔPITAL'S IS NOT THE QUOTIENT RULE!!

Find the following. Use L'Hôpital's when possible.

11. $\lim_{x\to 2}\frac{x-2}{x^2-7x+10} = \frac{0}{0}$

$\lim_{x\to 2}\frac{1}{2x-7}$

$\boxed{-\frac{1}{3}}$

12. $\lim_{x\to 0}\frac{3x^2}{e^x-1-x} = \frac{0}{0}$

$\lim_{x\to 0}\frac{6x}{e^x-1} = \frac{0}{0}$

$\lim_{x\to 0}\frac{6}{e^x} = \boxed{6}$

13. $\lim_{x\to 3}\frac{x^2-2x+1}{x-3} = \frac{4}{0}$

$\boxed{\text{Does not exist}}$

14. $\lim_{x\to 0}\frac{x^2}{1-\cos(3x)} = \frac{0}{0}$

$\lim_{x\to 0}\frac{2x}{3\sin(3x)} = \frac{0}{0}$

$\lim_{x\to 0}\frac{2}{9\cos(3x)} = \boxed{\frac{2}{9}}$

15. $\lim_{x\to 4}\frac{x^2+6x-40}{4-x} = \frac{0}{0}$

$\lim_{x\to 4}\frac{2x+6}{-1}$

$\boxed{-14}$

16. $\frac{d}{dx}\frac{3x-2}{5x+1}$ Quotient!

$\frac{3(5x+1)-(3x-2)(5)}{(5x+1)^2}$

$15x+3-15x+10$

$\boxed{\frac{13}{(5x+1)^2}}$