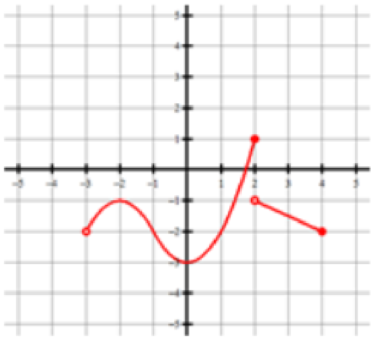
5.1 Extreme Values

PRACTICE

Find the extreme values and where they occur.

1.



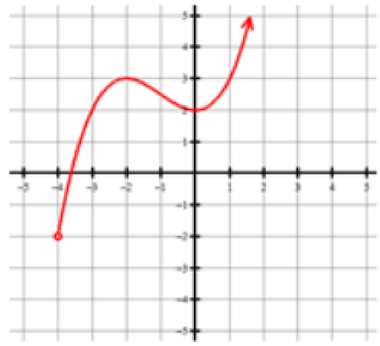
local max of -1 when $x = -2$

absolute min of -3 when $x = 0$

absolute max of 1 when $x = 2$

local min of -2 when $x = 4$

2.



local max of 3 when $x = -2$

local min of 2 when $x = 0$

3.

local min of -1 when $x = -5$

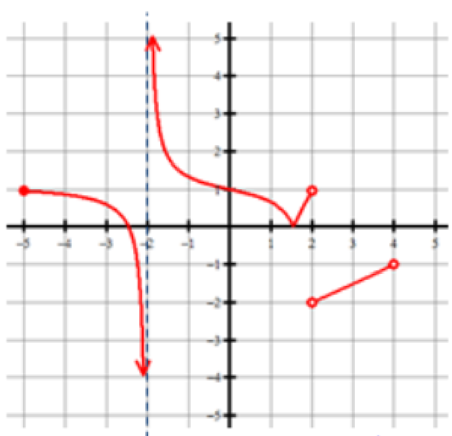
local max of 1 when $x = -1$

local min of 0 when $x = 0$

absolute max of 2 when $x = 2$

Use the graph of $f(x)$ to answer the following.

4.



Domain: $[-5,-2)(-2,2)(2,4)$

$\lim_{x\to 2^+} f(x) = -2$

$\lim_{x\to -2} f(x) =$ DNE

$\lim_{x\to 0} f(x) = 1$

$f(3) = -1.5$

$f'(3) = \frac{1}{2}$

Average rate of change over $[-5,-3]$

$(-5,1)\ (-3,0.5)$

$\frac{1-0.5}{-5--3} = \frac{0.5}{-2} = -\frac{1}{4}$

Absolute max: none

Absolute min: none

Local max: 1 when $x = -5$

Local min: 0 when $x = 1.5$

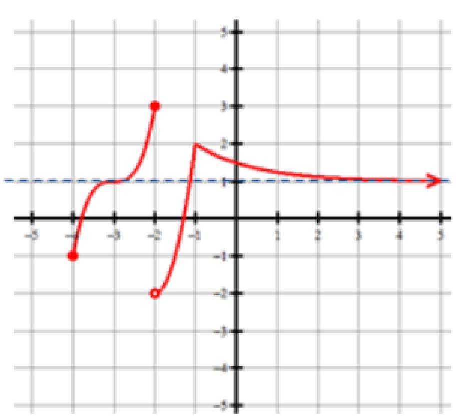
Interval(s) where $f(x)$ increasing

$(1.5,2)\ (2,4)$

Interval(s) where $f(x)$ decreasing

$(-5,-2)\ (-2,1.5)$

5.



Domain: $[-4,\infty)$

$\lim_{x\to -2^+} f(x) = -2$

$\lim_{x\to -2} f(x) =$ DNE

$\lim_{x\to \infty} f(x) = 1$

$f(-3) = 1$

$f'(-1) =$ DNE

Average rate of change over $[-4,-2]$

$(-4,-1)\ (-2,3)$

$\frac{-1-3}{-4--2} = \frac{-4}{-2} = 2$

Global max: 3 when $x = -2$

Global min: none

Relative max: 2 when $x = -1$

Relative min: -1 when $x = -4$

Interval(s) where $f(x)$ increasing

$(-4,-2)(-2,-1)$

Interval(s) where $f(x)$ decreasing

$(-1,\infty)$

Find the critical points.

6. $f(x) = 4x^3 - 9x^2 - 12x + 3$

$x = -\frac{1}{2}, 2$

7. $g(t) = \frac{2}{t^2-4}$ Note: $t \neq \pm 2$

$g(t) = 2(t^2-4)^{-1}$

$g'(t) = -2(t^2-4)^{-2}(2t)$

$0 = \frac{-4t}{(t^2-4)^2}$

$0 = -4t \qquad (t^2-4)^2 \neq 0$

$t = 0 \qquad \cancel{t = \pm 2}$

$t = \pm 2$ are not critical points because they are not in the domain of $g(t)$

8. $h(x) = \sqrt[3]{x-2}$

$x = 2$

9. $f(x) = (\ln x)^2$

$f'(x) = 2(\ln x)\frac{1}{x}$

$0 = \frac{2\ln x}{x}$

$\frac{0}{2} = \frac{2\ln x}{2} \qquad x \neq 0$

$\cancel{x = 0}$

$0 = \ln x$

$x = 0$ is not a critical point because it is not in the domain of $f(x)$

$e^0 = e^{\ln x}$

$1 = x$

10. $h(x) = 2\sin\left(\frac{x}{2}\right)$ where $-2\pi \le x \le 2\pi$

$h'(x) = 2\cos\left(\frac{1}{2}x\right)\left(\frac{1}{2}\right)$

$0 = \cos\left(\frac{1}{2}x\right)$

$\frac{1}{2}x = \frac{\pi}{2} \qquad \frac{1}{2}x = \frac{3\pi}{2}$

$x = \pi, -\pi$

11. $g(x) = e^x - x$

$g'(x) = e^x - 1$

$0 = e^x - 1$

$+1 \qquad +1$

$1 = e^x$

$\ln 1 = \ln e^x$

$\ln 1 = x$

$x = 0$

Find the absolute maximum and minimum values of the function on the given interval.

12. $f(x) = 1 + (x+1)^2, \quad [-2, 5]$

$f(-2) = 2$

$f(-1) = 1$ Absolute Min

$f(5) = 37$ Absolute Max

13. $f(x) = 2x^3 + 3x^2 + 4 \quad [-2, 1]$

$f'(x) = 6x^2 + 6x$

$0 = 6x^2 + 6x$

$0 = 6x(x+1)$

$x = 0, -1$

$f(-2) = 0$ Absolute Min

$f(-1) = 5$

$f(0) = 4$

$f(1) = 9$ Absolute Max

14. $f(x) = x^3 - 12x, \quad [0, 3)$

$f(0) = 0$ Absolute Max

$f(2) = -16$ Absolute Min

$x = 3$ is not included, can't be max/min

15. $h(x) = 3x^{\frac{2}{3}} - 2x, \quad [-1, 1]$

$h'(x) = 2x^{-1/3} - 2$

$0 = \frac{2}{\sqrt[3]{x}} - 2$

$\sqrt[3]{x} \cdot 2 = \frac{2}{\sqrt[3]{x}} \cdot \sqrt[3]{x} \qquad \sqrt[3]{x} \neq 0$

$\frac{2\sqrt[3]{x}}{2} = \frac{2}{2}$

$\sqrt[3]{x} = 1$

$x = 1, 0$

$f(-1) = 5$ Absolute Max

$f(0) = 0$ Absolute Min

$f(1) = 1$

Find the absolute maximum and minimum values of the function on the given interval.

16. $g(x) = x^2 + \frac{2}{x}$, $\left(\frac{1}{2}, 2\right]$

$x = \frac{1}{2}$ is not included, can't be max/min

$f(1) = 3$ Absolute Min

$f(2) = 5$ Absolute Max

17. $f(x) = \frac{x}{x^2+1}$, $[-2, 2]$

$u = x$ $\quad v = x^2 + 1$

$u' = 1$ $\quad v' = 2x$

$$f'(x) = \frac{(1)(x^2+1) - x(2x)}{(x^2+1)^2}$$

$$0 = \frac{-x^2+1}{(x^2+1)^2}$$

$0 = -x^2 + 1$ $\quad (x^2+1)^2 \neq 0$

$x = \pm 1$ $\quad$ DNE

$f(-2) = -\frac{2}{5}$

$f(-1) = -\frac{1}{2}$ Absolute Min

$f(1) = \frac{1}{2}$ Absolute Max

$f(2) = \frac{2}{5}$

18. $f(x) = \sin\left(x + \frac{\pi}{4}\right)$, $\left[0, \frac{7\pi}{4}\right]$

$f(0) = \frac{\sqrt{2}}{2}$

$f\left(\frac{\pi}{4}\right) = 1$ Absolute Max

$f\left(\frac{5\pi}{4}\right) = -1$ Absolute Min

$f\left(\frac{7\pi}{4}\right) = 0$

19. $g(x) = xe^{2x}$, $[-1, 1]$

$u = x$ $\quad v = e^{2x}$

$u' = 1$ $\quad v' = 2e^{2x}$

$g'(x) = (1)e^{2x} + x(2e^{2x})$

$0 = e^{2x} + 2xe^{2x}$

$0 = e^{2x}(1 + 2x)$

$x = -\frac{1}{2}$

$f(-1) = -e^{-2}$

$f\left(-\frac{1}{2}\right) = -\frac{1}{2}e^{-1}$ Absolute Min

$f(1) = e^2$ Absolute Max

TEST PREP

1. E
2. D
3. C
4. C
5. A
6. D