PRACTICE

Complete the sign chart and locate all extrema.

1. Given $f(x)$ is continuous and differentiable.

-2 0 3

Interval	$(-\infty,-2)$	$(-2,0)$	$(0,3)$	$(3,\infty)$
Test Value	$x=-4$	$x=-1$	$x=1$	$x=4$
$f'(x)$	$f(-4)=4$	$f(-1)=-3$	$f(1)=-7$	$f(4)=\frac{1}{2}$
Conclusion	increasing	decreasing	decreasing	increasing

max no etreme min

There is a relative maximum at x = -2 because f'(x) changes from positive to negative.
There is a relative minimum at x = 3 because f'(x) changes from negative to positive.

f'(x) does not change signs at x =0 therefore f has neither a relative max or min at x = 0

Use a sign chart to find the intervals where the function is increasing or decreasing and all extrema.

2. $f(x)=x^3-12x+1$

$f'(x)=3x^2-12$

$0=3(x^2-4)$

$0=3(x+2)(x-2)$

$x=-2,2$

-2 2

Interval	$(-\infty,-2)$	$(-2,2)$	$(2,\infty)$
Test Value	$x=-3$	$x=0$	$x=3$
$f'(x)$	$f'(-3)=15$	$f'(0)=-12$	$f'(3)=15$
Conclusion	increasing	decreasing	increasing

Max min

There is a relative maximum at x = -2 because f'(x) changes from positive to negative.
There is a relative minimum at x = 2 because f'(x) changes from negative to positive.

3. $g(x)=x^2(x-3)$ g'(x) = 0 when x = 0 and 2

g'(-1) = 9 positive g(x) is increasing

g'(1) = -3 negative g(x) is decreasing

There is a relative maximum at x = 0 because g'(x) changes from positive to negative.

g'(1) = -3 negative g(x) is decreasing

g'(3) = 9 positive g(x) is increasing

There is a relative minimum at x = 2 because g'(x) changes from negative to positive.

4. $f(x)=(x^2-1)^{\frac{2}{3}}$

$f'(x)=\frac{2}{3}(x^2-1)^{-1/3}(2x)$

$0=\frac{4x}{3\sqrt[3]{x^2-1}}$

$x=\pm 1 \quad x=0$

-1 0 1

Interval	$(-\infty,-1)$	$(-1,0)$	$(0,1)$	$(1,\infty)$
Test Value	$x=-2$	$x=-\frac{1}{2}$	$x=\frac{1}{2}$	$x=2$
$f'(x)$	$f'(-2)=\frac{-8}{3\sqrt[3]{3}}$	$f'\left(-\frac{1}{2}\right)=\frac{-2}{-3\sqrt[3]{\frac{3}{4}}}$	$f'\left(\frac{1}{2}\right)=\frac{2}{-3\sqrt[3]{\frac{3}{4}}}$	$f'(2)=\frac{8}{3\sqrt[3]{3}}$
Conclusion	decreasing	increasing	decreasing	increasing

Min Max Min

There is a relative minimum at x = -1 because f'(x) changes from negative to positive.
There is a relative maximum at x = 0 because f'(x) changes from positive to negative.
There is a relative minimum at x = 1 because f'(x) changes from negative to positive.

5. $g(t) = 12(1 + \cos t)$ on the interval $[0, 2\pi]$

$g'(x) = 0$ when $x = \pi$

There is a relative minimum at $t = \pi$ because $g'(t)$ changes from negative to positive.

$(0, \pi)$	$(\pi, 2\pi)$
$f'(x) < 0$ Decreasing	$f'(x) > 0$ Increasing

A particle moves along the x-axis with the position function given below. Find the velocity and use a sign chart to describe the motion of the particle.

6. $h(x) = -x^5 + \frac{5}{2}x^4 + \frac{40}{3}x^3 + 5$

$h'(x) = -5x^4 + 10x^3 + 40x^2$

$0 = -5x^2(x^2 - 2x - 8)$

$0 = -5x^2(x - 4)(x + 2)$

$x = 0, 4, -2$

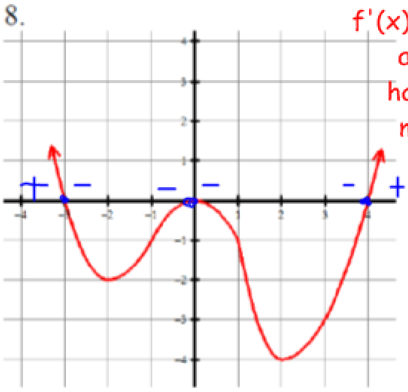
Interval	$(-\infty, -2)$	$(-2, 0)$	$(0, 4)$	$(4, \infty)$
Test Value	$x = -3$	$x = -1$	$x = 1$	$x = 5$
$f'(x)$	$f'(-3) = -315$	$f'(-1) = 25$	$f'(1) = 45$	$f'(5) = -875$
Conclusion	Decreasing Moving left	Increasing Moving right	Increasing Moving right	Decreasing Moving left

7. $g(x) = e^{\cos x}$ on the interval $[0, 2\pi]$

$[0, \pi)$	$(\pi, 2\pi]$
$f'(x) < 0$ Decreasing Moving left	$f'(x) > 0$ Increasing Moving right

Given the graph of $f'(x)$, find the critical points and locate all extrema.

8.

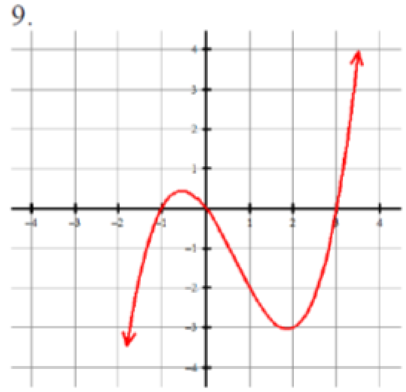


$f'(x)$ does not change signs at $x = 0$ therefore f has neither a relative max or min at $x = 0$

There is a relative maximum at $x = -3$ because $f'(x)$ changes from positive to negative.

There is a relative minimum at $x = 4$ because $f'(x)$ changes from negative to positive.

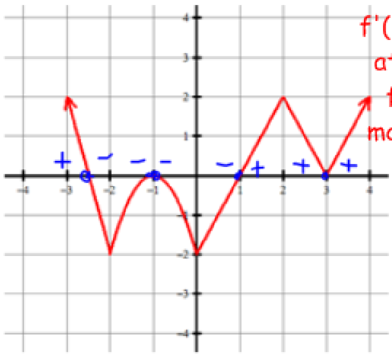
9.



There is a relative minimum at $x = -1$ and 3 because $f'(x)$ changes from negative to positive.

There is a relative maximum at $x = 0$ because $f'(x)$ changes from positive to negative.

10.

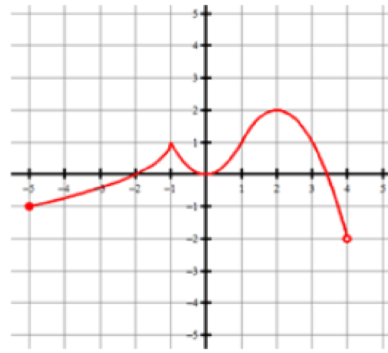


f'(x) does not change signs at x = -1 and 3 therefore f has neither a relative max or min at x = -1 and 3

There is a relative maximum at x = -2.5 because f'(x) changes from positive to negative.

There is a relative minimum at x = 1 because f'(x) changes from negative to positive.

11.



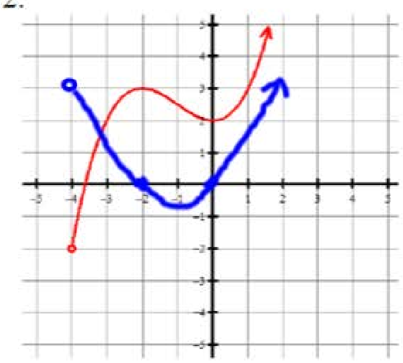
f'(x) does not change signs at x = 0 therefore f has neither a relative max or min at x = 0

There is a relative minimum at x = -2 because f'(x) changes from negative to positive.

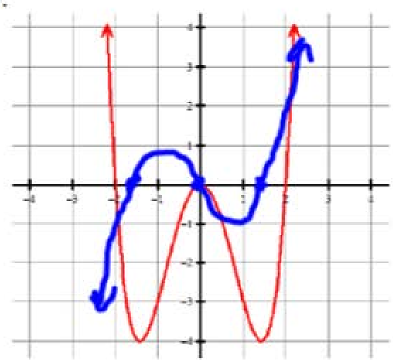
There is a relative maximum at x = 3.5 because f'(x) changes from positive to negative.

Given the $f(x)$, sketch an approximate graph of $f'(x)$.

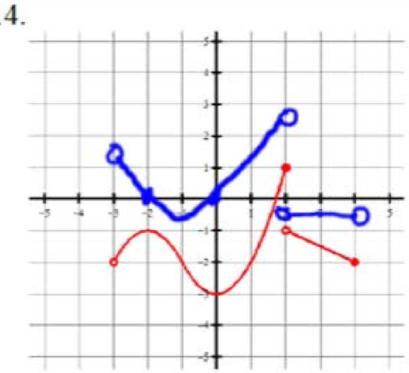
12.



13.



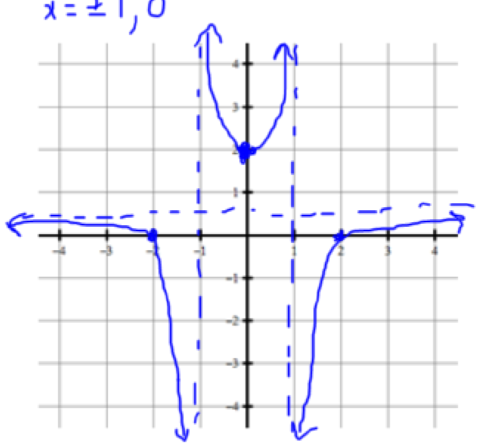
14.



15. $f(x) = \frac{x^2-4}{2x^2-2}$

$f'(x) = \frac{12x}{(2x^2-2)^2}$

$x = \pm 1, 0$



Domain: $(-\infty,-1)(-1,1)(1,\infty)$

Horizontal Asymptote(s):

$\lim_{x\to\infty} f(x) = 1/2$

$\lim_{x\to-\infty} f(x) = 1/2$

Vertical Asymptote(s): $x = \pm 1$

x-intercept(s): $(-2,0)\ (2,0)$

y-intercept: $(0,2)$

Make sign chart showing:
- Extrema
- Interval(s) where $f(x)$ increasing
- Interval(s) where $f(x)$ decreasing

x = 0 relative minimum

$-1 \qquad 0 \qquad 1$

Interval	$(-\infty,-1)$	$(-1,0)$	$(0,1)$	$(1,\infty)$
Test Value	$x=-2$	$x=-\frac{1}{2}$	$x=\frac{1}{2}$	$x=2$
$f'(x)$	$f'(-2)=-\frac{2}{3}$	$f'\left(-\frac{2}{3}\right)=-3$	$f'\left(\frac{1}{2}\right)=\frac{2}{3}$	$f'(2)=\frac{1}{3}$
Conclusion	decreasing	decreasing	increasing	increasing

no etreme   min   no extreme

Multiple Choice

1. B
2. B
3. A
4. B
5. D

Free Response

a. The phrase "farthest to the left" tells us we are looking for the minimum position. To find minimum position, we start by seeing where the derivative of position (which is velocity) equals zero or fails to exist: $x'(t) = v(t) = t^3 - 6t^2 = 0$ for $t = 0$ or $t = 6$.

Time interval	$0 < t < 6$	$6 < t \le 10$
Sign of velocity	negative	positive
Direction of movement	left	right

1 point critical points

1 point justification

Since the particle moves to the left for $0 < t < 6$ (indicated by negative velocity) and moves to the right for $6 < t \le 10$, it is farthest to the left at $t = 6$.

b. To find where velocity is increasing the fastest, we really are looking for the maximum acceleration. Acceleration is given by $a(t) = 3t^2 - 12t$. Since $a'(t) = 6t - 12 = 0$ for $t = 2$ only, the only critical point is at $t = 2$. Compare the values of $a(t)$ at the critical point and endpoints:

t	$a(t)$
0	0
2	-12
10	180

1 point critical point

1 point justification

Therefore, velocity is increasing the fastest at $t = 10$.