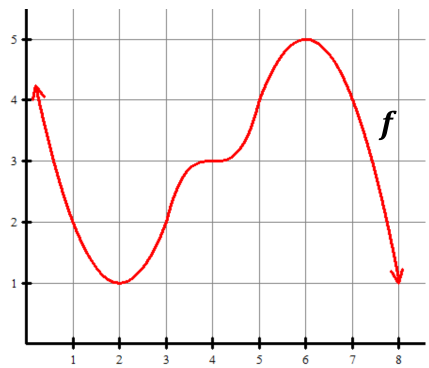
5.3 Second Derivative Test

NOTES

CALCULUS

Write your questions here!



CONCAVITY

Point of Inflection

Interval	$(-\infty, 3)$	$(3, 4)$	$(4, 5)$	$(5, \infty)$
$f''(x)$				
Conclusion				

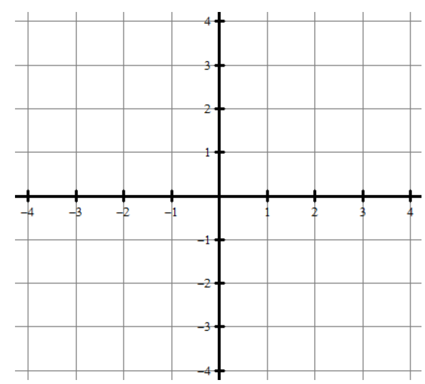
Second Derivative Test

Suppose $f'(c) = 0$. Then:

- If $f''(c) > 0$, then f has a relative minimum at $x = c$.
- If $f''(c) < 0$, then f has a relative maximum at $x = c$.

Use the second derivative test to find the relative extrema.

$f(x) = x^4 - 2x^2$



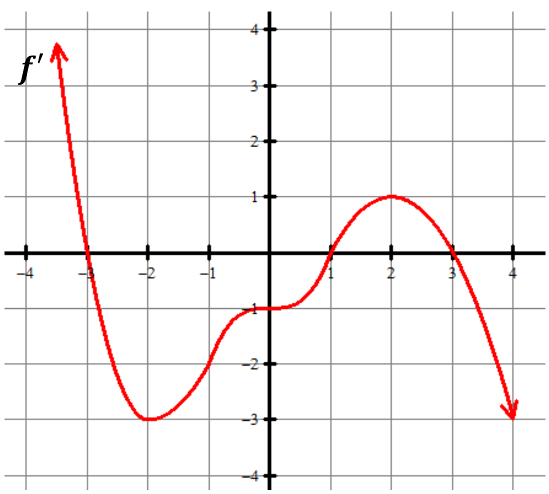
Do a sign analysis of second derivative to find intervals where *f* is concave up or down.

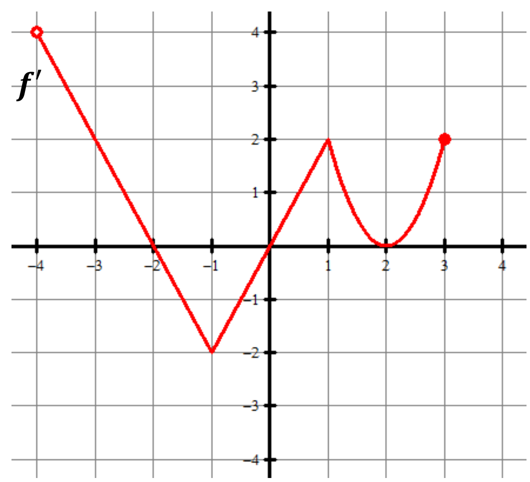
Interval			
Test Value			
$f''(x)$			
Conclusion			

Particle Motion

A particle is moving along the *x*-axis with position function $x(t) = \frac{1}{3}t^3 - 4t^2 + 12t$. Find the velocity and acceleration. Describe the motion of the particle.

Given the graph of f', find the points of inflection and state the intervals of concavity.





SUMMARY:

Now, summarize your notes here!

Use the sign chart(s) to answers the following.

1. Given $g(x)$ is twice differentiable on $[-3, 3]$

x	$-3 < x < -2$	-2	$-2 < x < 1$	1	$1 < x < 3$
$g'(x)$	Negative	0	Positive	0	Negative

x	$-3 < x < -\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2} < x < 3$
$g''(x)$	Positive	0	Negative

Intervals where $g(x)$ is increasing:

Intervals where $g(x)$ is decreasing:

Extrema:

Intervals where $g(x)$ is concave up:

Intervals where $g(x)$ is concave down:

Points of Inflection:

2. Given $f(x)$ is continuous and twice differentiable.

Interval	$x < -1$	$x = -1$	$-1 < x < 1$	$x = 1$	$1 < x < 2$	$x = 2$	$2 < x < 4$	$x = 4$	$4 < x < 5$	$x = 5$	$x > 5$
$f'(x)$	Positive	0	Negative	Negative	Negative	0	Negative	Negative	Negative	0	Positive
$f''(x)$	Negative	Negative	Negative	0	Positive	0	Negative	0	Positive	Positive	Positive

Intervals where $f(x)$ is increasing:

Intervals where $f(x)$ is decreasing:

Extrema:

Intervals where $f(x)$ is concave up:

Intervals where $f(x)$ is concave down:

Points of Inflection:

3. Given $f(x)$ is continuous and twice differentiable.

Interval	$(-\infty, -2)$	-2	$(-2, 3)$	3	$(3, \infty)$
Test Value	$x = -4$	$x = -2$	$x = 0$	$x = 3$	$x = 4$
$f'(x)$	$f'(-4) = 4$	$f'(-2) = 0$	$f'(0) = -7$	$f'(3) = -3$	$f'(4) = -4$
$f''(x)$	$f''(-4) = -6$	$f''(-2) = -4$	$f''(0) = -7$	$f''(3) = 0$	$f''(4) = 8$

Intervals where $f(x)$ is increasing:

Intervals where $f(x)$ is decreasing:

Extrema:

Intervals where $f(x)$ is concave up:

Intervals where $f(x)$ is concave down:

Points of Inflection:

Find the points of inflection.

4. $f(x) = \sin\frac{x}{2}$ on the interval $(-\pi, 3\pi)$

5. $f(x) = e^{-x^2}$

Find all points of inflection and relative extrema. Use the Second Derivative Test where applicable.

6. $f(x) = 5 + 3x^2 - x^3$

7. $h(x) = (2x - 5)^2$

8. $f(x) = x + 2\sin x$ on the interval $(0, 2\pi)$

9. $f(x) = 2x^4 - 8x + 3$

State the intervals of concavity.

10. $g(x) = \frac{x}{x-1}$

11. $f(x) = x^3 - 12x$

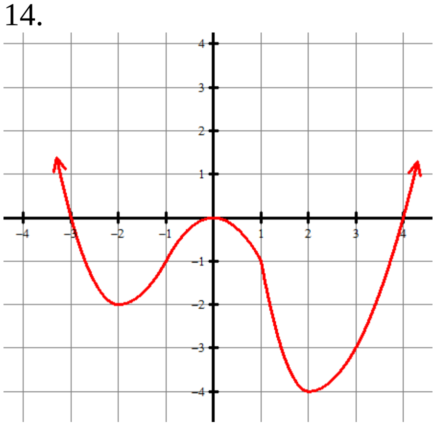
A particle moves along the *x*-axis with the position function given below. Find the velocity and acceleration. Use a sign chart to describe the motion of the particle.

12. $x(t) = \frac{1}{3}t^3 - 3t^2 + 8t + 1$ where $t > 0$

13. $x(t) = t - 3(t-4)^{\frac{1}{3}}$ where $t > 0$

Given the graph of $f'(x)$. State the intervals of concavity. Find the point(s) of inflection.

14.



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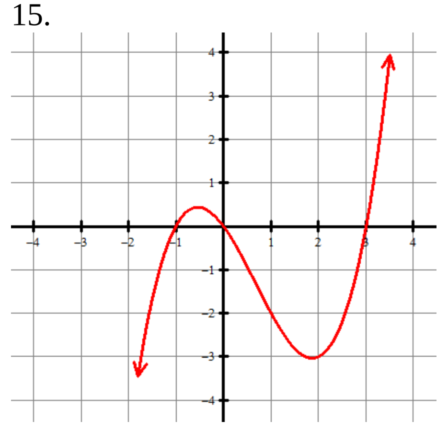

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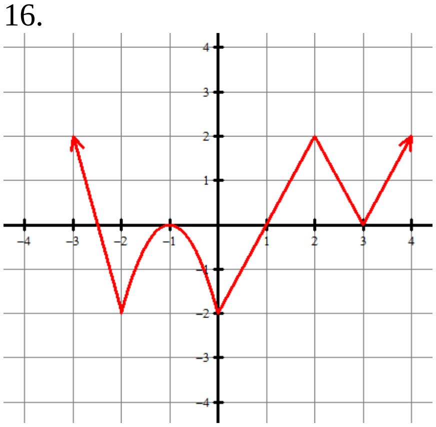

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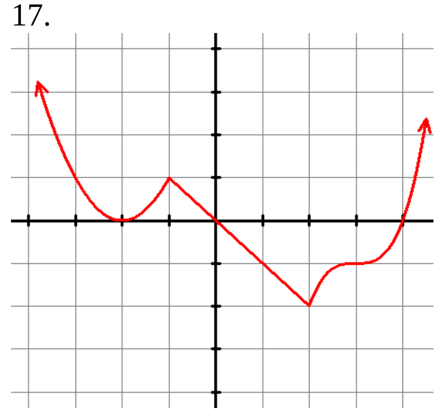

MULTIPLE CHOICE

1. Find the point of inflection of $g(x) = x^2 - \frac{8}{x}$ when $x > 0$.

    (A) 1
    (B) 2
    (C) 4
    (D) 8
    (E) 16

2. The domain of the function f is $x > 0$. If $f'(x) = x \ln x$, then $f(x)$ is concave down for all

    (A) $0 < x < 1$

    (B) $0 < x < e$

    (C) $0 < x < \frac{1}{e}$

    (D) $x > \frac{1}{e}$

    (E) $x > e$

3. Consider a function f whose first derivative is given by $f'(x) = \frac{1 - \ln x}{x^2}$. It is clear that $f'(e) = 0$, so e is a critical number. The value $f''(e)$ is

    (A) negative, making $f(e)$ a local minimum

    (B) positive, making $f(e)$ a local minimum

    (C) negative, making $f(e)$ a local maximum

    (D) positive, making $f(e)$ a local maximum

    (E) none of the above

4. Consider the function given by $f(x) = 27x - x^3$. The function f is decreasing on the interval(s)

    (A) $[-3, 3]$ only

    (B) $[0, 3]$ only

    (C) $[0, \infty)$ only

    (D) $\left[-3\sqrt{3}, 3\sqrt{3}\right]$ only

    (E) $(-\infty, -3]$ and $[3, \infty)$

5. Let f be a function defined for all real numbers x. If $f'(x) = \frac{|9-x^2|}{x-3}$, then f is decreasing on the interval

(A) $(-\infty, 3)$

(B) $(-\infty, \infty)$

(C) $(-3, 6)$

(D) $(-3, \infty)$

(E) $(3, \infty)$

 **You are allowed to use a graphing calculator** 

FREE RESPONSE

Your score: ____ out of 7

1. A particle moves along a straight line. For $0 \le t \le 5$, the velocity of the particle is given by $v(t) = -2 + (t^2 + 3t)^{6/5} - t^3$, and the position of the particle is given by $s(t)$. It is known that $s(0) = 10$.

   a. Find all values of t in the interval $2 \le t \le 4$ for which the speed of the particle is 2.

   b. Find all times t in the interval $0 \le t \le 5$ at which the particle changes direction. Justify your answer.

   c. Is the speed of the particle increasing or decreasing at time $= 4$? Give a reason for your answer.