Use the sign chart(s) to answers the following.

1. Given $g(x)$ is twice differentiable on $[-3, 3]$

x	$-3 < x < -2$	-2	$-2 < x < 1$	1	$1 < x < 3$
$g'(x)$	Negative	0	Positive	0	Negative

min max

x	$-3 < x < -\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2} < x < 3$
$g''(x)$	Positive	0	Negative

Intervals where $g(x)$ is increasing: $(-2, 1)$

Intervals where $g(x)$ is decreasing: $(-3, -2)$ $(1, 3)$

Extrema: $x = -2$ relative min

$x = 1$ relative max

There is a relative minimum at x = -2 because f'(x) changes from negative to positive.

There is a relative maximum at x = 1 because f'(x) changes from positive to negative.

Intervals where $g(x)$ is concave up: $(-3, -\frac{1}{2})$

Intervals where $g(x)$ is concave down: $(-\frac{1}{2}, 3)$

Points of Inflection: $x = -\frac{1}{2}$

f''(x) = 0

f''(x) changes positive to negative.

2. Given $f(x)$ is continuous and twice differentiable.

Interval	$x < -1$	$x = -1$	$-1 < x < 1$	$x = 1$	$1 < x < 2$	$x = 2$	$2 < x < 4$	$x = 4$	$4 < x < 5$	$x = 5$	$x > 5$
$f'(x)$	Positive	max 0	Negative	Negative	Negative	0	Negative	Negative	Negative	min 0	Positive
$f''(x)$	Negative	Negative	Negative	0	Positive	0	Negative	0	Positive	Positive	Positive

Intervals where $f(x)$ is increasing: $(-\infty, -1)$ $(5, \infty)$

Intervals where $f(x)$ is decreasing: $(-1, 5)$

Extrema: $x = -1$ relative max

$x = 5$ relative min

There is a relative maximum at x = -1 because f'(x) changes from positive to negative.

There is a relative minimum at x = 5 because f'(x) changes from negative to positive.

Intervals where $f(x)$ is concave up: $(1, 2)$ $(4, \infty)$

Intervals where $f(x)$ is concave down: $(-\infty, 1)$ $(2, 4)$

Points of Inflection: $x = 1$ $x = 4$

$x = 2$

f''(x) = 0

f''(x) changes signs

3. Given $f(x)$ is continuous and twice differentiable.

Interval	$(-\infty,-2)$	-2	$(-2,3)$	3	$(3,\infty)$
Test Value	$x=-4$	$x=-2$	$x=0$	$x=3$	$x=4$
$f'(x)$	$f'(-4)=4$	$f'(-2)=0$	$f'(0)=-7$	$f'(3)=-3$	$f'(4)=-4$
$f''(x)$	$f''(-4)=-6$	$f''(-2)=-4$	$f''(0)=-7$	$f''(3)=0$	$f''(4)=8$

Intervals where $f(x)$ is increasing: $(-\infty,-2)$

Intervals where $f(x)$ is decreasing: $(-2,\infty)$

Extrema: $x=-2$ relative max

There is a relative maximum at x = -2 because f'(x) changes from positive to negative.

Intervals where $f(x)$ is concave up: $(3,\infty)$

Intervals where $f(x)$ is concave down: $(-\infty,3)$

Points of Inflection: $x=3$

f''(x) = 0

f''(x) changes negative to positive

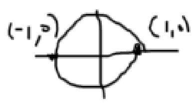
Find the points of inflection.

4. $f(x)=\sin\frac{x}{2}$ on the interval $(-\pi, 3\pi)$

$f'(x)=\frac{1}{2}\cos\left(\frac{x}{2}\right)$

$f''(x)=-\frac{1}{4}\sin\left(\frac{x}{2}\right)$

$-\frac{1}{4}\sin\left(\frac{x}{2}\right)=0$



$\frac{x}{2}=0$ $\quad \frac{x}{2}=\pi$

$x=0$ $\quad x=2\pi$

test	$-\frac{\pi}{2}$	0	π	2π	$\frac{5\pi}{2}$
$f''(x)$	$\frac{\sqrt{2}}{8}$	0	$-\frac{1}{4}$	0	$\frac{\sqrt{2}}{8}$

$x=0$ is point of inflection

$f''(0)=0$, $f''\left(-\frac{\pi}{2}\right)>0$

$f''(\pi)<0$

f''(x) changes signs from positive to negative

$x=2\pi$ is a point of inflection

$f''(2\pi)=0$, $f''(\pi)<0$, $f''\left(\frac{5\pi}{2}\right)>0$

f''(x) changes signs from negative to positive

5. $f(x)=e^{-x2}$

test	-1	$-\frac{\sqrt{2}}{2}$	0	$\frac{\sqrt{2}}{2}$	1
$f''(x)$	$2e^{-1}$	0	-2	0	$2e^{-1}$

$f'(x)=-2xe^{-x^2}$

$f''(x)=4x^2e^{-x^2}-2e^{-x^2}$

$2e^{-x^2}(2x^2-1)=0$

$2e^{-x^2}\neq 0$ $\quad$ $2x^2-1=0$, $2x^2=1$, $x^2=\frac{1}{2}$ $\quad x=\pm\sqrt{\frac{1}{2}}=\pm\frac{1}{\sqrt{2}}=\pm\frac{\sqrt{2}}{2}$

$x=-\frac{\sqrt{2}}{2}$ is a point of inflection

$f''\left(-\frac{\sqrt{2}}{2}\right)=0$, $f''(-1)>0$, $f''(0)<0$

f''(x) changes signs from positive to negative

$x=\frac{\sqrt{2}}{2}$ is a point of inflection

$f''\left(\frac{\sqrt{2}}{2}\right)=0$, $f''(0)<0$, $f''(1)>0$

f''(x) changes signs from negative to positive

Find all points of inflection and relative extrema. Use the Second Derivative Test where applicable.

6. $f(x) = 5 + 3x^2 - x^3$

$x = 0$ relative min $f''(0) = 6$

$f''(0) > 0$ concave up

$x = 2$ relative max $f''(2) = -6$

$f''(2) < 2$ concave down

$x = 1$ point of inflection

$f''(1) = 0$, $f''(0) > 0$

$f''(2) < 0$

f''(x) changes signs from positive to negative

7. $h(x) = (2x - 5)^2$

$h'(x) = 2(2x-5)(2)$

$0 = 8x - 20$

$x = \frac{20}{8} = \frac{5}{2}$

$h''(x) = 8$

$x = \frac{5}{2}$ relative min

$f''(\frac{5}{2}) = 8$

$f''(\frac{5}{2}) > 0$

Concave up

No points of inflection

8. $f(x) = x + 2\sin x$ on the interval $[0, 2\pi]$

$x = \frac{2\pi}{3}$ relative max $f''\left(\frac{2\pi}{3}\right) = -\sqrt{3}$

$f''(\frac{2\pi}{3}) < 0$ concave down

$x = \frac{4\pi}{3}$ relative min $f''(\frac{4\pi}{3}) = \sqrt{3}$

$f''(\frac{4\pi}{3}) > 0$ concave up

$x = \pi$ point of inflection

$f''(\pi) = 0$, $f''(\frac{2\pi}{3}) < 0$

$f''(\frac{4\pi}{3}) > 0$

f''(x) changes signs from negative to positive

Note: 0 and 2π would be points of inflection if they weren't the endpoints. Can't be a point of inflection at endpoint bc you aren't changing concavity!

9. $f(x) = 2x^4 - 8x + 3$

$f'(x) = 8x^3 - 8$

$0 = 8x^3 - 8$

$8 = 8x^3$

$1 = x^3$

$1 = x$

$f''(x) = 24x^2$

$0 = 24x^2$

$0 = x$ Not a point of inflection

$f''(-1) = 24$

$f''(1) = 24$

Does not change concavity!

$x = 1$ relative min

$f''(1) = 24$

$f''(1) > 0$

concave up

State the intervals of concavity.

10. $g(x) = \frac{x}{x-1}$

$(-\infty, 1)$ concave down $f''(x) < 0$

$(1, \infty)$ concave up $f''(x) > 0$

11. $f(x) = x^3 - 12x$

$f'(x) = 3x^2 - 12$

$f''(x) = 6x$

$x = 0$ point of inflection

test	-1	0	1
$f''(x)$	-6	0	6

Concave down $(-\infty, 0)$

Concave up $(0, \infty)$

A particle moves along the x-axis with the position function given below. Find the velocity and acceleration. Use a sign chart to describe the motion of the particle.

12. $x(t) = \frac{1}{3}t^3 - 3t^2 + 8t + 1$ where $t > 0$

$v(t) = x'(t) = t^2 - 6t + 8$

$a(t) = x''(t) = 2t - 6$

Interval	$(0,2)$	2	$(2,3)$	3	$(3,4)$	4	$(4,\infty)$
Test Value	$t = 1$	$t = 2$	$t = \frac{5}{2}$	$t = 3$	$t = \frac{7}{2}$	$t = 4$	$t = 5$
$f'(x)$	$f'(1) = 3$ Increasing right	$f'(2) = 0$ No Velocity	$f'\left(\frac{5}{2}\right) = -\frac{3}{4}$ Decreasing left	$f'(3) = -1$ Decreasing left	$f'\left(\frac{7}{2}\right) = -\frac{3}{4}$ Decreasing left	$f'(4) = 0$ No Velocity	$f'(5) = 3$ Increasing right
$f''(x)$	$f''(1) = -4$	$f''(0) = -6$ (2) = -2	$f''\left(\frac{5}{2}\right) = -1$	$f''(3) = 0$	$f''\left(\frac{7}{2}\right) = 1$	$f''(4) = 2$	$f''(5) = 4$
Conclusion	Slowing down	Not moving	Speeding Up		Slowing down	Not moving	Speeding up

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13. $x(t) = t - 3(t-4)^{\frac{1}{3}}$ where $t > 0$

$v(t) = x'(t) = 1 - (t-4)^{-2/3}$

$0 = 1 - \frac{1}{\sqrt[3]{(t-4)^2}}$ $t \neq 4$

$\frac{1}{\sqrt[3]{(t-4)^2}} = 1$

$1 = \sqrt[3]{(t-4)^2}$

$1^3 = (t-4)^2$

$\pm\sqrt{1} = t - 4$

$4 \pm 1 = t$

$t = 5$ or 3

$a(t) = x''(t) = \frac{2}{3}(t-4)^{-5/3}$

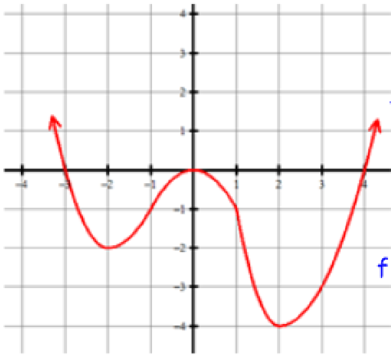
$0 = \frac{2}{3\sqrt[3]{(t-4)^5}}$

$t \neq 4$

Interval	$(0,3)$	3	$(3,4)$	4	$(4,5)$	5	$(5,\infty)$
Test Value	$t = 1$	$t = 3$	$t = \frac{7}{2}$	$t = 4$	$t = \frac{9}{2}$	$t = 5$	$t = 6$
$f'(x)$	$f'(1) > 0$ Increasing right	$f'(3) = 0$ No Velocity	$f'\left(\frac{7}{2}\right) < 0$ Decreasing left	$f'(4) = \text{DNE}$ No Velocity	$f'\left(\frac{9}{2}\right) < 0$ Decreasing left	$f'(5) = 0$ No Velocity	$f'(6) > 0$ Increasing right
$f''(x)$	$f''(1) < 0$	$f''(3) < 0$	$f''\left(\frac{7}{2}\right) < 0$	$f''(4) = \text{DNE}$	$f''\left(\frac{9}{2}\right) > 0$	$f''(5) > 0$	$f''(6) > 0$
Conclusion	Slowing down	Not moving	Speeding up	Not moving	Slowing down	Not moving	Speeding up

Given the graph of $f'(x)$, find the points of inflection.

14.



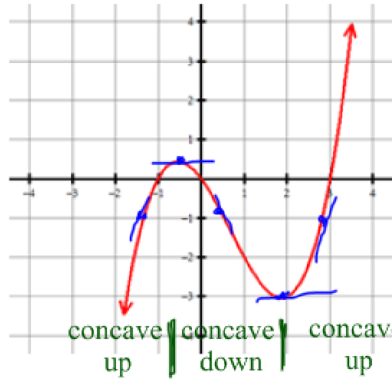
$x = -2, 0, 2$

f''(-2) changes signs from negative to positive

f''(0) changes signs from positive to negative

f''(2) changes signs from negative to positive

15.

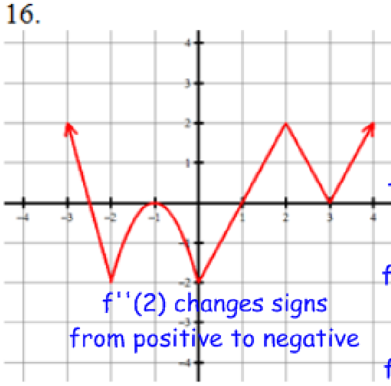


$x \approx -0.5, 1.9$

f''(-0.5) changes signs from positive to negative

f''(1.9) changes signs from negative to positive

16.



$x = -2, -1, 0, 2, 3$

f''(-2) changes signs from negative to positive

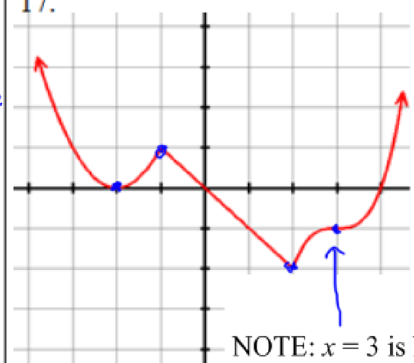
f''(-1) changes signs from positive to negative

f''(0) changes signs from negative to positive

f''(2) changes signs from positive to negative

f''(3) changes signs from negative to positive

17.



$x = -2, -1, 2$

f''(-2) changes signs from negative to positive

f''(-1) changes signs from positive to negative

f''(2) changes signs from negative to positive

NOTE: $x = 3$ is NOT a point of inflection
While $f''(x) = 0$ indicates it could be,
the function does not change concavity!

TEST PREP

MULTIPLE CHOICE

1. B
2. C
3. C
4. E
5. A

FREE RESPONSE 7 points

(a) Solve $\lvert v(t)\rvert = 2$ on $2 \le t \le 4$. $t = 3.128$ (or 3.127) and $t = 3.473$	$2: \begin{cases} 1: \text{considers } \lvert v(t)\rvert = 2 \\ 1: \text{answer} \end{cases}$
(b) $v(t) = 0$ when $t = 0.536033,\ 3.317756$ $v(t)$ changes sign from negative to positive at time $t = 0.536033$. $v(t)$ changes sign from positive to negative at time $t = 3.317756$. Therefore, the particle changes direction at time $t = 0.536$ and time $t = 3.318$ (or 3.317).	$3: \begin{cases} 1: \text{considers } v(t) = 0 \\ 2: \text{answers with justification} \end{cases}$
(c) $v(4) = -11.475758 < 0$, $a(4) = v'(4) = -22.295714 < 0$ The speed is increasing at time $t = 4$ because velocity and acceleration have the same sign.	2 : conclusion with reason