Calculus

Write your questions and thoughts here!

6.1 Implicit Differentiation

Name:__________________________

Notes

Recall:

Explicit equation | Implicit equation

Finding the derivative **explicitly**: $y^2 + 3x = 5x^3$

When you can't isolate y in terms of x (or if solving for y makes taking the derivative CRAZY), then you want to take the derivative implicitly.

Implicit Differentiation Example: Find $\frac{dy}{dx}$ for $y^2 + 3x = 5x^3$

Step 1: Take the derivative normally. Each time a "y" is involved, include a $\frac{dy}{dx}$.

Step 2: Gather all terms with $\frac{dy}{dx}$ on the left side, everything else on the right.

Step 3: Factor out the $\frac{dy}{dx}$ if necessary to create only one $\frac{dy}{dx}$ term.

Step 4. Solve for $\frac{dy}{dx}$.

2. $y^3 - 2x = x^4 + 2y$

3. $3x^2 + 4xy^2 - 5y^3 = 10$

6.1 Implicit Differentiation

Notes

Write your questions and thoughts here!

Derivative at a point – implicit differentiation.

4. Find the equation of any tangent line for $x^2 + y^2 = 4$ at $x = 1$.

2nd Derivative – Implicit Differentiation:

Finding the 2nd derivative implicitly is a little trickier than finding it explicitly. Once you have done a few, you'll see it's just a matter of algebraic substitution.

5. Find $\frac{d^2y}{dx^2}$ for $\cos y = 2x^2$

Now summarize what you learned!

6.1 Implicit Differentiation

Calculus Name: ______________________________ **Practice**

Find $\frac{dy}{dx}$.

1. $4 = 5x^2 + 2y^3$	2. $5y^2 + 3 = x^2$	3. $3x = y^3 + 4$
4. $x^2 = 4y^3 + 5y^2$	5. $(4y^3 + 4)^2 = 3x^2$	6. $2x^3 = (3y^3 + 4)^2$
7. $-3y + y^3 = 5x$	8. $5x^3 - 2y = 5y^3$	9. $\sin(x + y) = 2x$
10. $4x + 1 = \cos y^2$	11. $3x^2 - 6y^2 + 5 = 9y - 3x$	12. $y^2 - 7y + x^2 - 4x = 10$

13. $e^{y^3} = x^3 + 1$

14. $5x^2 - e^{4y^2} = -6$

15. $\ln(4y^3) = 5x + 3$

16. $x^3 + 1 = \ln(3y^7)$

17. $x^3 + y^3 = 6xy$

18. $x^3 - 3x^2y^2 = 3y^3$

For 19-23, use implicit differentiation to find $\frac{d^2y}{dx^2}$.

19. $xy = -3$

20. $x^2 + y^2 = 8$

21. $y^2 = 5x^2 - 3x$

22. $y^3 = x^2 - 4$

23. $y^2 + 3y = 4x - 5$

Find the slope of the tangent line at the given point.

24. $2 = 3x^4 + xy^4$ at $(-1, 1)$

25. $x^2 - y^2 = 27$ at $(6, -3)$

26. $x \ln y = 4 - 2x$ at $(2, 1)$

27. $(x - y)^2 - 4x = 20y$ at $(4, 2)$

Write an equation of the line tangent to the curve at the given point.

28. $x^2 + y^2 + 19 = 2x + 12y$ at $(4, 3)$

29. $x \sin 2y = y \cos 2x$ at $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

30. Find the points on the curve $x^2 + 2y^2 = 8$ where the tangent line is parallel to the x-axis.

31. Find the point(s) where the following graph has a **vertical** tangent line. $x + y = y^2$

6.1 Implicit Differentiation

Test Prep

1. If $x + \sin y = \ln y$, then $\frac{dy}{dx} =$

(A) $y + y\cos y$ (B) $\frac{y+\cos y-1}{y}$ (C) $\frac{1-y}{y\cos y}$

(D) $\frac{y}{y\cos y+1}$ (E) $\frac{y}{1-y\cos y}$

---

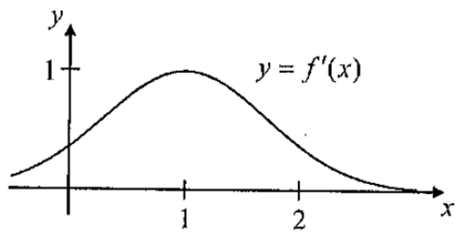
2. The first derivative of the function f is given by $f'(x) = \frac{\cos^2 x}{x} - \frac{1}{5}$. How many critical values does f have on the open interval $(0, 10)$?

(A) One (B) Three (C) Four (D) Five (E) Seven

---

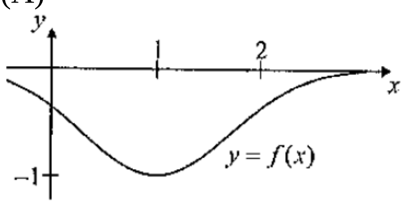
3. A curve is generated by the equation $x^2 + 4y^2 = 16$. Determine the number of points on this curve whose corresponding tangent lines are horizontal.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

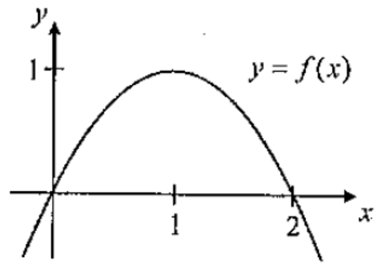


4. The graph of $f'(x)$ is shown above. Which of the following could be the graph of $f(x)$?

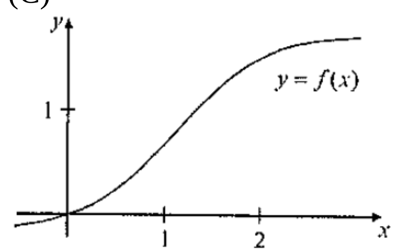
(A)



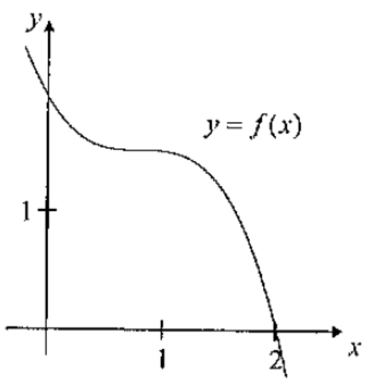
(B)



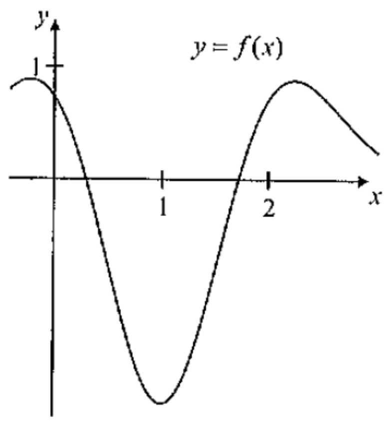
(C)



(D)



(E)



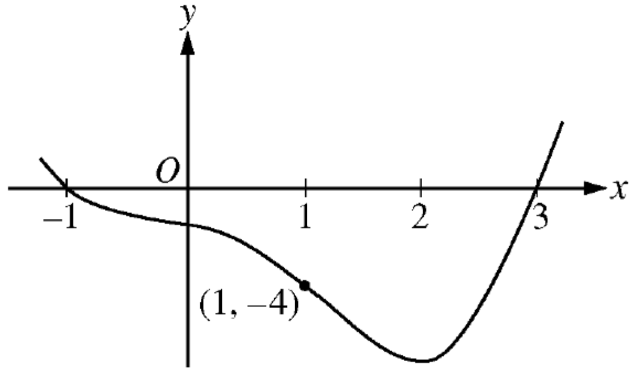
---

5. A curve given by the equation $x^3 + xy = 8$ has slope given by $\frac{dy}{dx} = \frac{-3x^2 - y}{x}$. The value of $\frac{d^2y}{dx^2}$ at the point where $x = 2$ is

(A) -6 (B) -3 (C) 0 (D) 4 (E) undefined

FREE RESPONSE **Your score: _______ out of 9**

2009 Form B AB5



Graph of f'

Let f be a twice-differentiable function defined on the interval $-1.2 < x < 3.2$ with $f(1) = 2$. The graph of f', the derivative of f, is shown above. The graph of f' crosses the x-axis at $x = -1$ and $x = 3$ and has a horizontal tangent at $x = 2$. Let g be the function given by $g(x) = e^{f(x)}$.

(a) Write an equation for the line tangent to the graph of g at $x = 1$.

(b) For $-1.2 < x < 3.2$, find all values of x at which g has a local maximum. Justify your answer.

(c) The second derivative of g is $g''(x) = e^{f(x)}\left[\left(f'(x)\right)^2 + f''(x)\right]$. Is $g''(-1)$ positive, negative or zero? Justify your answer.

(d) Find the average rate of change of g', the derivative of g, over the interval [1,3].