Name: Solutions   Date: _____ Period: _____

6 Review – Implicit Differentiation

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you must study all packets from Unit 6.

6.1 Implicit Differentiation

Step 1: Take the derivative normally. Each time a "y" is involved, include a $\frac{dy}{dx}$.

Step 2: Gather all terms with $\frac{dy}{dx}$ on the left side, everything else on the right.

Step 3: If necessary, factor out the $\frac{dy}{dx}$ to create only one $\frac{dy}{dx}$ term.

Step 4. Solve for $\frac{dy}{dx}$.

Find $\frac{dy}{dx}$.

1. $8 = 3x^2 + y^4$

$0 = 6x + 4y^3 \frac{dy}{dx}$

$\frac{-6x}{4y^3} = \frac{dy}{dx}$

$$\boxed{\frac{dy}{dx} = -\frac{3x}{2y^3}}$$

2. $\sin(2x - y) = 4x$

$\cos(2x - y) \cdot (2 - \frac{dy}{dx}) = 4$

$2 - \frac{dy}{dx} = \frac{4}{\cos(2x-y)}$

$$\boxed{\frac{dy}{dx} = 2 - \frac{4}{\cos(2x-y)}}$$

3. $x^2 + 2y^5 = 10xy$

$2x + 10y^4 \frac{dy}{dx} = 10y + 10x\frac{dy}{dx}$

$10y^4\frac{dy}{dx} - 10x\frac{dy}{dx} = 10y - 2x$

$\frac{dy}{dx}(10y^4 - 10x) = 10y - 2x$

$$\boxed{\frac{dy}{dx} = \frac{5y - x}{5y^4 - 5x}}$$

Use implicit differentiation to find $\frac{d^2y}{dx^2}$.

4. $7xy = 8$

$7y + 7x\frac{dy}{dx} = 0$

$7x\frac{dy}{dx} = -7y$

$\frac{dy}{dx} = \boxed{-\frac{y}{x}}$

$-\frac{\frac{dy}{dx} \cdot x - y}{x^2}$

$-\frac{-\frac{y}{x} \cdot x - y}{x^2}$

$\frac{d^2y}{dx^2} = \boxed{\frac{2y}{x^2}}$

5. $e^x + y^2 = 4$

$e^x + 2y\frac{dy}{dx} = 0$

$2y\frac{dy}{dx} = -e^x$

$\frac{dy}{dx} = \boxed{-\frac{e^x}{2y}}$

$-\frac{e^x(2y) - e^x \cdot 2\frac{dy}{dx}}{4y^2}$

$-\frac{2ye^x - 2e^x\left(-\frac{e^x}{2y}\right)}{4y^2}$
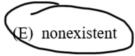
$\frac{d^2y}{dx^2} = \boxed{-\frac{2ye^x + \frac{e^{2x}}{y}}{4y^2}}$

6. If $x^2y + y^2 + 4 = 0$, then when $x = 2$, the value of $\frac{dy}{dx}$ is

E

(A) -2 (B) -1 (C) 0 (D) 2 (E) nonexistent

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7. If $x^2 - y^2 = 5$, what is the value of $\frac{d^2y}{dx^2}$ at the point $(3, 2)$?

D

(A) $-\frac{13}{8}$ (B) $-\frac{11}{8}$ (C) $-\frac{7}{8}$ (D) $-\frac{5}{8}$ (E) $-\frac{1}{4}$

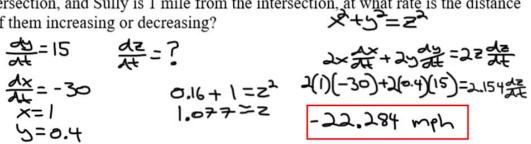## 6.2 Related Rates

Guidelines to solving related rate problems
1. Draw a picture.
2. Make a list of all known and unknown rates and quantities.
3. Relate the variables in an equation.
4. Differentiate with respect to time.
5. Substitute the known quantities and rates and solve.

8. Brust is riding his bicycle north away from an intersection at a rate of 15 miles per hour. Sully is driving his car towards the intersection from the west at a rate of 30 miles per hour. If Brust is 0.4 miles from the intersection, and Sully is 1 mile from the intersection, at what rate is the distance between the two of them increasing or decreasing?

$x^2 + y^2 = z^2$

$\frac{dy}{dt} = 15$ $\frac{dz}{dt} = ?$ $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$\frac{dx}{dt} = -30$ $0.16 + 1 = z^2$ $2(1)(-30) + 2(0.4)(15) = 2.154\frac{dz}{dt}$

$x = 1$ $1.077 \approx z$

$y = 0.4$

$\boxed{-22.284 \text{ mph}}$

9. The side of a cube is increasing at a constant rate of 0.2 centimeters per second. In terms of the surface area S, what is the rate of change of the volume of the cube, in cubic centimeters per second?
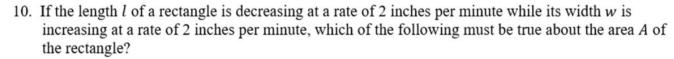
A

(A) $0.1S$ (B) $0.2S$ (C) $0.6S$

(D) $0.04S$ (E) $0.008S$

10. If the length l of a rectangle is decreasing at a rate of 2 inches per minute while its width w is increasing at a rate of 2 inches per minute, which of the following must be true about the area A of the rectangle?
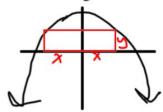
C

(A) A is always increasing.   (B) A is always decreasing.   (C) A is increasing only when $l > w$.
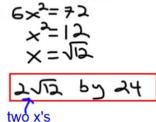
(D) A is increasing only when $l < w$.   (E) A remains constant.

6.3 Optimization

Strategies for solving optimization problems:
1. Draw a picture (if applicable) and identify *known* and *unknown* quantities.
2. Write an equation (model) that will be optimized.
3. Write your equation in terms of a single variable.
4. Determine the desired max or min value with calculus techniques.
5. Determine the domain (endpoints) of your equation to verify if the endpoints represent a max or min.

11. A rectangle is formed with the base on the x-axis and the top corners on the function $y = 36 - x^2$. What length and width should the rectangle have so that its area is a maximum?

$A = 2x \cdot y$
$A = 2x(36 - x^2)$
$A = 72x - 2x^3$
$A' = 72 - 6x^2$
$0 = 72 - 6x^2$

$6x^2 = 72$
$x^2 = 12$
$x = \sqrt{12}$

$2\sqrt{12}$ by 24

two x's

12. The minimum acceleration attained on the interval $0 \le t \le 4$ by the particle whose velocity is given by $v(t) = t^3 - 4t^2 - 3t + 2$ is

D

(A) -16 (B) -10 (C) -8

(D) $-\dfrac{25}{3}$ (E) -3