7.2 Trapezoidal Approximation

NOTES

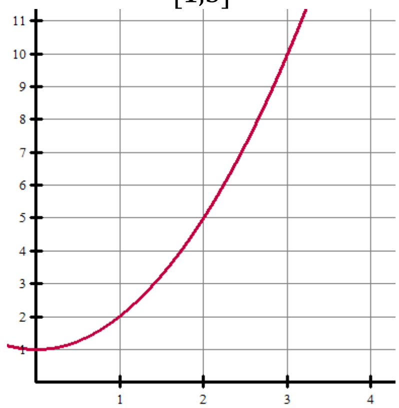
CALCULUS

Write your questions here!

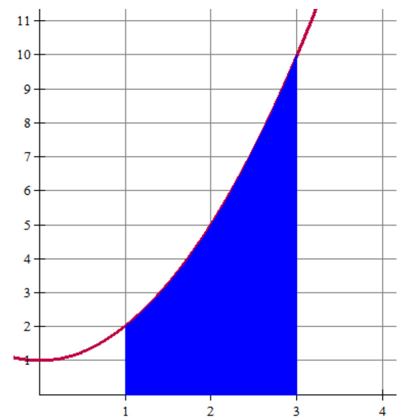
Riemann Sum

$\boldsymbol{f(x) = x^2 + 1}$

[1,3]



$\boldsymbol{f(x) = x^2 + 1}$



The Definite Integral

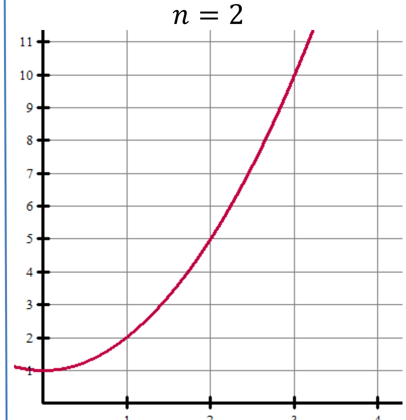
$$\int_a^b f(x)dx$$

Trapezoidal Approximation for interval [1,3] with *n* subintervals

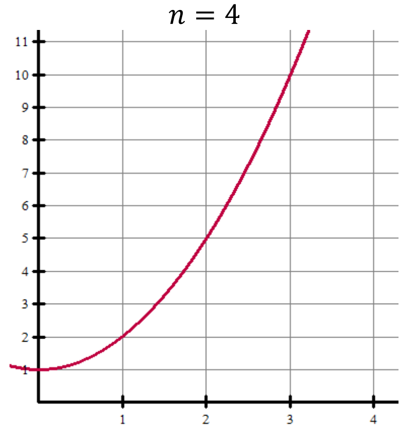
How close is it the true value?

$\boldsymbol{f(x) = x^2 + 1}$

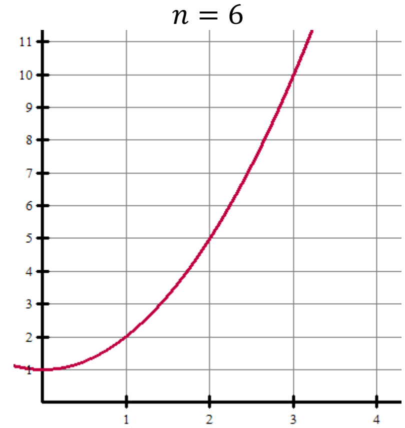
$n = 2$



$n = 4$



$n = 6$



Given the definite integral $\int_{-1}^{2}(10 - x^2)dx$

(a) Use the Trapezoid Rule with three equal subintervals to approximate its value.

(b) Is your answer from part (a) an overestimate or underestimate?

(c) Calculate the exact value on your calculator. Compare to your answer in part (a).

The rate at which customers are being served at Starbrusts is given by the continuous $R(t)$. A table of selected values of $R(t)$, for the time interval $0 < t < 10$ hours, is given below. At $t = 0$ there had been 200 customers served.

Time (hours)	0	2	3	6	10
$R(t)$ (people/hour)	37	44	36	42	48

Use a trapezoidal sum with 4 subintervals to approximate $\int_{0}^{10} R(t)dt$

Approximately how many customers had been served after 10 hours?

SUMMARY:

Now, summarize your notes here!

7.2 Trapezoidal Approximation

PRACTICE

 You can use a calculator on 1-13 

Sketch the trapezoidal approximations. Find the width of each subinterval. Write a definite integral.

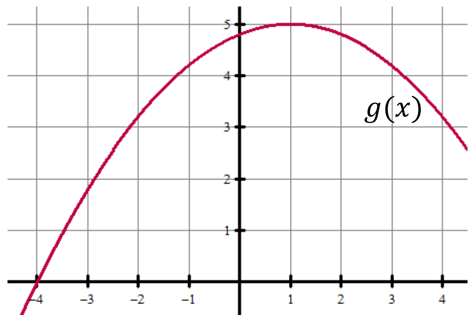
1. $n = 6$ subintervals on [1,4]

Width of each subinterval =

Definite Integral =

2. $n = 4$ subintervals on [-3,0]

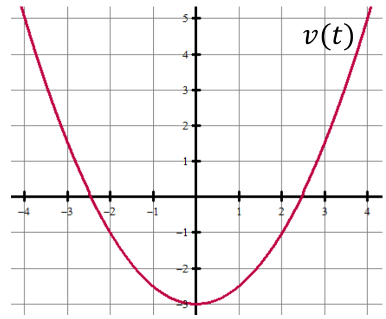
Width of each subinterval =



Definite Integral =

3. $n = 10$ subintervals on [-1,4]

Width of each subinterval =



Definite Integral =

Approximate the definite integral use trapezoidal approximation.

4. $\int_{-2}^{4} (9 + x^2)dx$

Trapezoidal approximation with 6 subintervals

5. $\int_{0}^{2} \left(\frac{4 - x}{x + 5}\right) dx$

Trapezoidal approximation with 4 subintervals

6. $\int_{1}^{2} (3\sqrt{x + 2})dx$

Trapezoidal approximation with 3 subintervals

Use the calculator to find the exact value of the definite integral.

7. $\int_{1}^{3} \left(\frac{1}{5}x^3 - 2x^2 + 10\right) dx$

8. $\int_{-2}^{0} (e^x + 4)dx$

9. $\int_{\pi}^{\frac{3\pi}{2}} (\sin^2 x)dx$

10. $\int_{-2}^{e} \left(4 - \sqrt{9 - x^2}\right) dx$

Use the information provided to answer the following.

11. Let $y(t)$ represent the rate of change of the population of a town over a 20-year period, where y is a differentiable function of t. The table shows the population change in people per year recorded at selected times. The population at $t = 0$ was 25,500 people.

Time (years)	0	4	10	13	20
$\boldsymbol{y(t)}$ **(people per year)**	2500	2724	3108	3697	4283

a. Use a trapezoidal approximation with four subintervals to approximate $\int_0^{20} y(t)dt$

b. What is the approximate population after 20 years?

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12. The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by the twice-differentiable and strictly increasing function R of time t. A table of selected values of $R(t)$ for the time interval $0 \leq t \leq 90$ minutes is shown below. At $t = 0$ the plane had already consumed 84 gallons of fuel.

Time (minutes)	0	30	40	50	70	90
$\boldsymbol{R(t)}$ **(gallons per min)**	20	30	40	55	65	70

a. Use data from the table to find an approximation for $R'(45)$. Show the computations that led to your answer. Indicate units of measure.

b. Approximate the value of $\int_0^{90} R(t)dt$ using trapezoidal approximation with five subintervals indicated by the data in the table.

c. Approximately how much fuel has the plane consumed after 90 minutes?

You can use a calculator on 1-4

MULTIPLE CHOICE

1. If three equal subdivisions of $[-2,4]$ are used, what is the trapezoidal approximation of $\int_{-2}^{4} \frac{e^x}{2} dx$?
   (A) $e^4 + e^2 + e^0 + e^{-2}$
   (B) $e^4 + 2e^2 + 2e^0 + e^{-2}$
   (C) $\frac{1}{2}(e^4 + e^2 + e^0 + e^{-2})$
   (D) $\frac{1}{2}(e^4 + 2e^2 + 2e^0 + e^{-2})$
   (E) $\frac{1}{4}(e^4 + 2e^2 + 2e^0 + e^{-2})$

2. Suppose that $f'(x) > 0$ on $(-\infty, -1)$, $f'(x) < 0$ on $(-1,1)$ and $f'(x) > 0$ on $(1, \infty)$. Which of the following functions could be f?
   (A) $f(x) = x^2 - 2x$
   (B) $f(x) = x^3 - 3x$
   (C) $f(x) = x^2 + 2x$
   (D) $f(x) = x^4 - 4x$
   (E) $f(x) = x^3 + 3x$

3. A continuous function $f(x)$ has the values shown in the table. What is the value of a trapezoidal approximation of $\int_0^3 f(x)$ using six equal subintervals?

x	0	0.5	1.0	1.5	2.0	2.5	3.0
$f(x)$	8	5	4	3	3	5	8

   (A) 9
   (B) 14
   (C) 18
   (D) 28
   (E) 56

4. Determine the slope of the normal line to the curve $x^3 + xy^2 = 10y$ at the point $(2,1)$.
   (A) 0
   (B) 2
   (C) $-\frac{7}{3}$
   (D) $-\frac{6}{13}$
   (E) $\frac{1}{2}$

CALCULATOR ACTIVE

FREE RESPONSE

Your score: ____ out of 6

A test plane flies in a straight line with positive velocity $v(t)$, in miles per minute at time *t* minutes where *v* is a differentiable function of *t*. Selected values of $v(t)$ for $0 \leq t \leq 40$ are shown in the table below.

***t* (min)**	0	5	10	15	20	25	30	35	40
$v(t)$ (mpm)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3

(a) Use a midpoint Riemann sum with four subintervals of equal length and values from the table to approximate $\int_0^{40} v(t)\,dt$. Show the computations that lead to your answer. Using correct units, explain the meaning of $\int_0^{40} v(t)\,dt$ in terms of the plane's flight.

(b) Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval $0 < t < 40$? Justify your answer.

(c) The function *f*, defined by $f(t) = 6 + \cos\left(\frac{t}{10}\right) + 3\sin\left(\frac{7t}{40}\right)$, is used to model the velocity of the plane, in miles per minute, for $0 \leq t \leq 40$. According to this model, what is the acceleration of the plane at $t = 23$? Indicate units of measure.