7.2 Trapezoidal Approximation

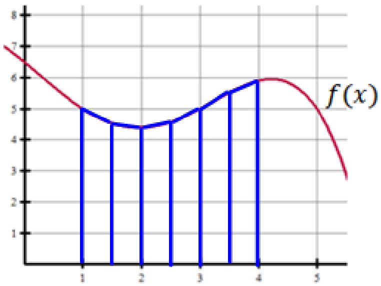
PRACTICE

You can use a calculator on 1-13

Sketch the trapezoidal approximations. Find the width of each subinterval. Write a definite integral.

1. $n = 6$ subintervals on[1,4]

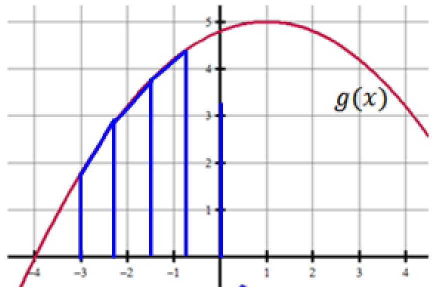
Width of each subinterval = $\frac{4-1}{6} = \frac{1}{2}$



Definite Integral = $\int_1^4 f(x)\,dx$

2. $n = 4$ subintervals on[-3,0]

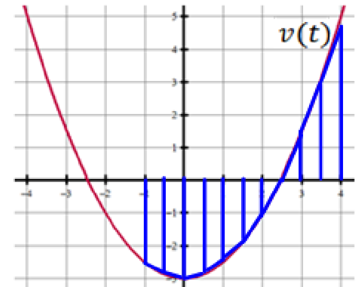
Width of each subinterval = $\frac{3}{4}$



Definite Integral = $\int_{-3}^{0} g(x)\,dx$

3. $n = 10$ subintervals on[-1,4]

Width of each subinterval = $\frac{4-{-1}}{10} = \frac{1}{2}$



Definite Integral = $\int_{-1}^{4} v(t)\,dt$

Approximate the definite integral use trapezoidal approximation.

4. $\int_{-2}^{4} (9 + x^2)dx$

Trapezoidal approximation with 6 subintervals

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5. $\int_{0}^{2} \left(\frac{4-x}{x+5}\right)dx$ $\quad \frac{2-0}{4} = \frac{2}{4} = \frac{1}{2}$

Trapezoidal approximation with 4 subintervals

$\frac{1}{2}\left[\frac{f(0)+f(0.5)}{2} + \frac{f(0.5)+f(1)}{2} + \frac{f(1)+f(1.5)}{2} + \frac{f(1.5)+f(2)}{2}\right]$

$f(0) = 0.8$
$f(0.5) = 0.63636$
$f(1) = 0.5$
$f(1.5) = 0.38462$
$f(2) = 0.28571$

$1.03\overline{2}$

6. $\int_{1}^{2} (3\sqrt{x+2})dx$ $\quad \frac{2-1}{3} = \frac{1}{3}$

Trapezoidal approximation with 3 subintervals

5.607

Use the calculator to find the exact value of the definite integral.

7. $\int_{1}^{3} \left(\frac{1}{5}x^3 - 2x^2 + 10\right) dx$

$6.\overline{6}$

8. $\int_{-2}^{0} (e^x + 4)dx$

8.8646

9. $\int_{\pi}^{\frac{3\pi}{2}} (\sin^2 x)dx$

0.785

10. $\int_{-2}^{e} \left(4 - \sqrt{9 - x^2}\right) dx$

6.525

Use the information provided to answer the following.

11. Let $y(t)$ represent the rate of change of the population of a town over a 20-year period, where y is a differentiable function of t. The table shows the population change in people per year recorded at selected times. The population at $t = 0$ was 25,500 people.

Time (years)	0	4	10	13	20
$y(t)$ (people per year)	2500	2724	3108	3697	4283

a. Use a trapezoidal approximation with four subintervals to approximate $\int^{20} y(t)\,dt$

$$4\left(\frac{2500+2724}{2}\right)+6\left(\frac{2724+3108}{2}\right)+3\left(\frac{3108+3697}{2}\right)+7\left(\frac{3697+4283}{2}\right)$$

$$4(2612)+6(2916)+3(3402.5)+7(3990)$$

66081.5

b. What is the approximate population after 20 years?

$66081.5 + 25500 =$ 91581.5 people

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12. The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by the twice-differentiable and strictly increasing function R of time t. A table of selected values of $R(t)$ for the time interval $0 \le t \le 90$ minutes is shown below. At $t = 0$ the plane had already consumed 84 gallons of fuel.

Time (minutes)	0	30	40	50	70	90
$R(t)$ (gallons per min)	20	30	40	55	65	70

a. Use data from the table to find an approximation for $R'(45)$. Show the computations that led to your answer. Indicate units of measure.

$$\frac{55-40}{50-40} = \frac{3}{2} \text{ gallons/min}^2$$

b. Approximate the value of $\int_0^{90} R(t)dt$ using trapezoidal approximation with five subintervals indicated by the data in the table.

4125

c. Approximately how much fuel has the plane consumed after 90 minutes?

4209 gallons of fuel

MULTIPLE CHOICE

1. D
2. B
3. B
4. D

CALCULATOR ACTIVE

FREE RESPONSE

Your score: ____ out of 6

(a) Midpoint Riemann sum is

$$10 \cdot [v(5) + v(15) + v(25) + v(35)]$$
$$= 10 \cdot [9.2 + 7.0 + 2.4 + 4.3] = 229$$

The integral gives the total distance in miles that the plane flies during the 40 minutes.

$$3 : \begin{cases} 1 : v(5) + v(15) + v(25) + v(35) \\ 1 : \text{answer} \\ 1 : \text{meaning with units} \end{cases}$$

(b) By the Mean Value Theorem, $v'(t) = 0$ somewhere in the interval $(0, 15)$ and somewhere in the interval $(25, 30)$. Therefore the acceleration will equal 0 for at least two values of t.

$$2 : \begin{cases} 1 : \text{two instances} \\ 1 : \text{justification} \end{cases}$$

(c) $f'(23) = -0.407$ or -0.408 miles per minute^2

1 : answer with units