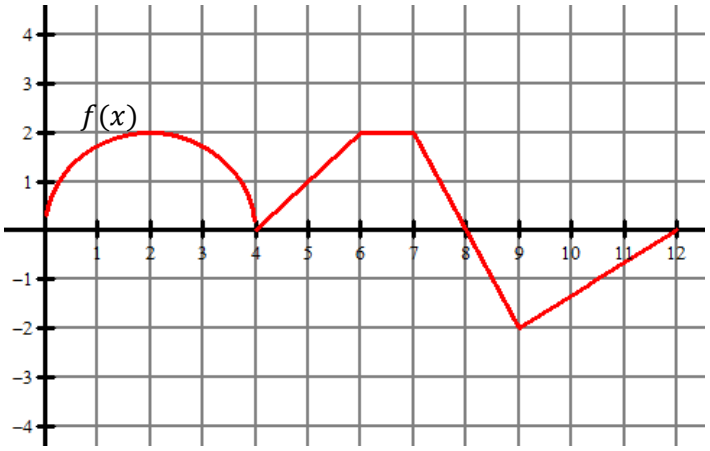
8.1 Definite Integral

NOTES

CALCULUS

Write your questions here!



$\int_0^4 f(x)dx =$

$\int_4^8 f(x)dx =$

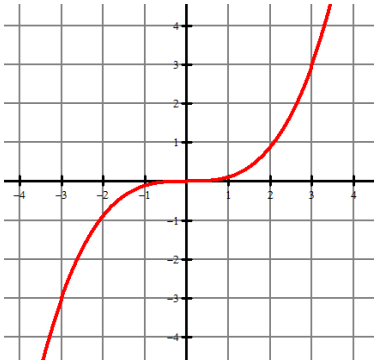
$\int_8^{12} f(x)dx =$

$\int_0^8 f(x)dx$

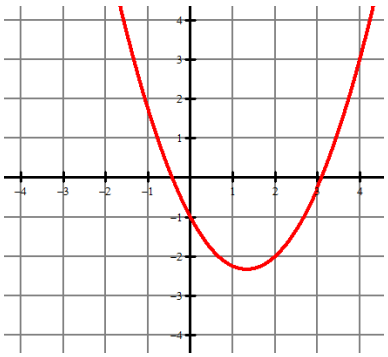
$\int_0^{12} f(x)dx$

Set up the integral for the following:

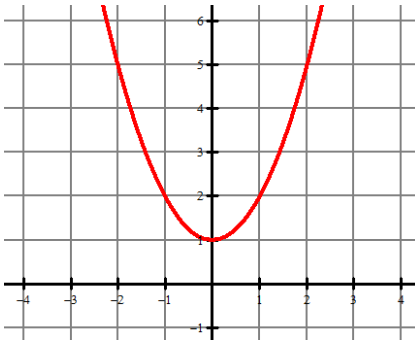
$f(x) = \frac{1}{9}x^3$



$y = \frac{3}{4}x^2 - 2x - 1$

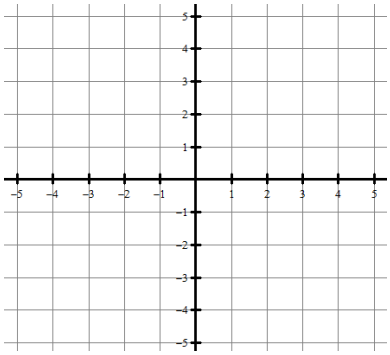


$y = x^2 + 1$

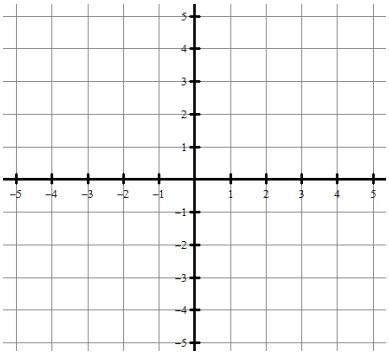


Sketch a graph of the definite integral. Use the calculator to evaluate.

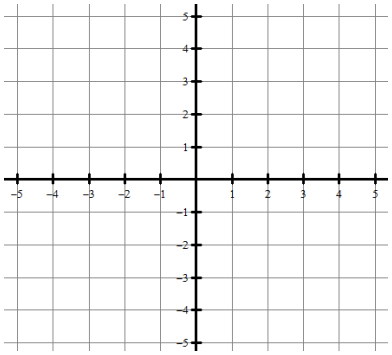
$\int_{-2}^{1} (4 - x^2)\,dx$



$\int_2^3 \sqrt{x-1}\,dx$



$\int_{-2}^{4} \left(\frac{x}{3} - 1\right)dx$



Properties of Definite Integrals

Zero Integral	Negation
Multiply by constant (k = constant)	Decomposition ($a < c < b$)
Addition	Subtraction

Given

$$\int_{-2}^{1} f(x)dx = 4 \qquad \int_{1}^{5} f(x)dx = -3 \qquad \int_{-2}^{1} g(x)dx = 8$$

Find the following (if possible)

$(a)\ \int_{5}^{1} f(x)dx$	$(b)\ \int_{-2}^{5} f(x)dx$	$(c)\ \int_{-2}^{1} [f(x) + 2g(x)]dx$
$(d)\ \int_{0}^{1} f(x)dx$	$(e)\ \int_{1}^{-2} 3f(x)dx$	$(f)\ \int_{5}^{5} [f(x) - g(x)]dx$

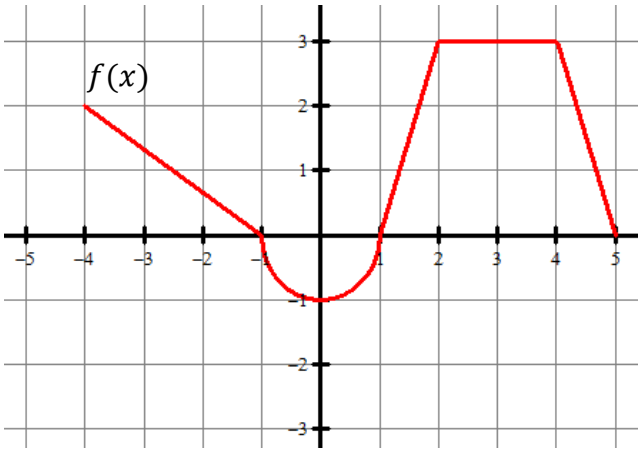
SUMMARY:

Now, summarize your notes here!

PRACTICE

The graph of *f* consists of line segments and a semicircle. Evaluate each definite integral.

1.



(a) $\int_{-4}^{-1} f(x)dx =$

(b) $\int_{1}^{2} f(x)dx =$

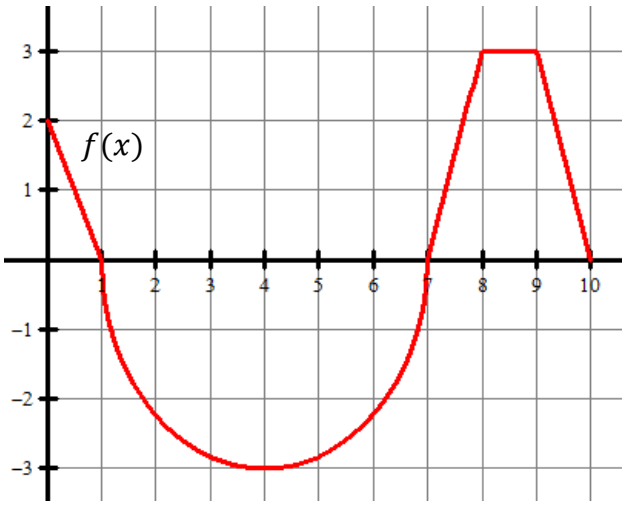
(c) $\int_{1}^{5} f(x)dx =$

(d) $\int_{-4}^{5} f(x)dx =$

(e) $\int_{4}^{2} f(x)dx =$

(f) $\int_{-1}^{1} f(x)dx =$

2.



(a) $\int_{0}^{1} 2f(x)dx =$

(b) $\int_{1}^{7} f(x)dx =$

(c) $\int_{0}^{7} f(x)dx =$

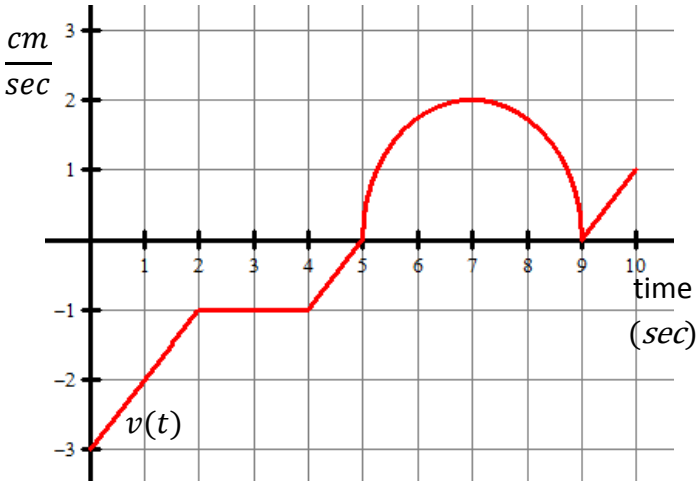
(d) $\int_{10}^{7} f(x)dx =$

(e) $\int_{8}^{8} f(x)dx =$

(f) $\int_{0}^{10} f(x)dx =$

The velocity of a particle moving along the *x*-axis is graphed with line segments and a semi-circle below.

3.



(a) Find $\int_0^{10} v(t)dt$. What does it represent?

(b) What is the total distance travelled?

(c) When is the particle speeding up?

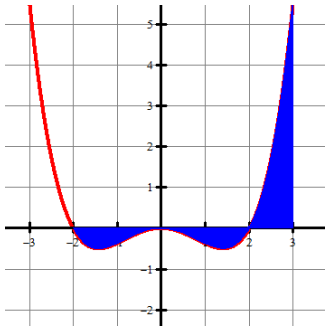
(d) When is the particle slowing down?

(e) How much does the particle move left?

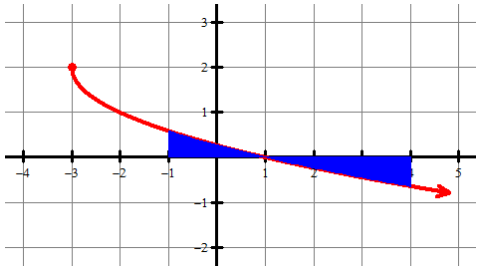
(f) What is happening at $t = 7$?

Set up a definite integral to represent the following.

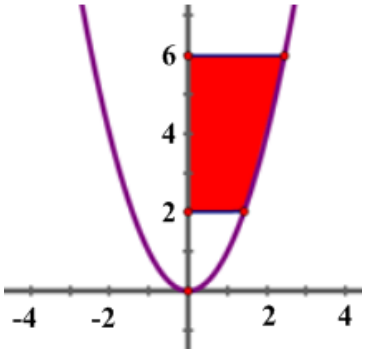
4. $f(x) = \frac{x^4}{8} - \frac{x^2}{2}$



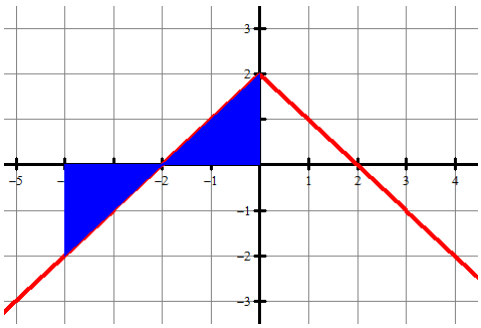
5. $y = -\sqrt{x+3} + 2$



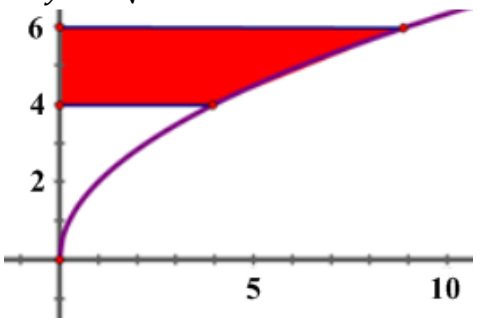
6. $y = x^2$



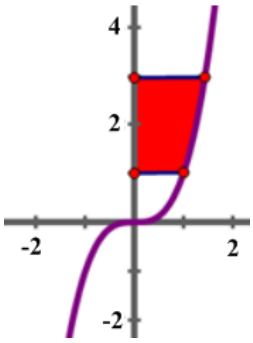
7. $f(x) = -|x| + 2$



8. $y = 2\sqrt{x}$



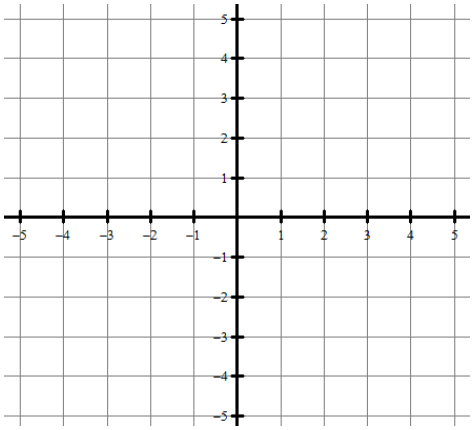
9. $y = x^3$



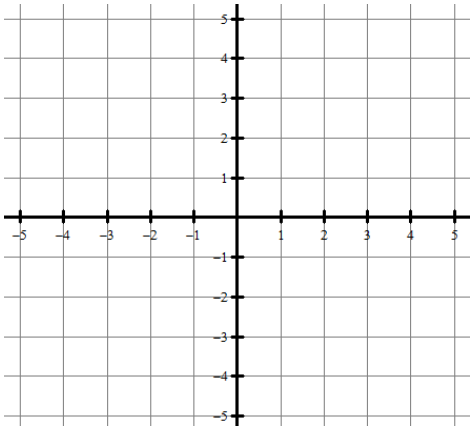
USE A GRAPHING CALCULATOR ON 10-28

Sketch a graph of the definite integral without the calculator. Evaluate with the graphing calculator.

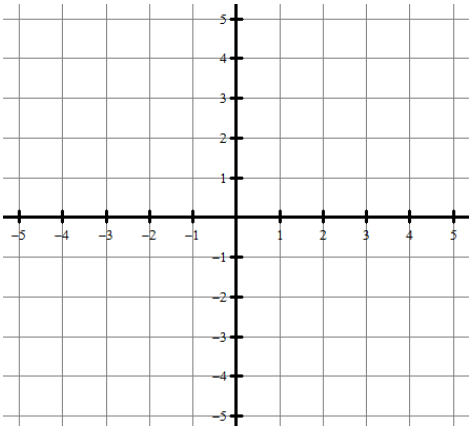
10. $\int_0^3 -\sqrt{x+1}\,dx =$



11. $\int_{-2}^{1} (x^2 - 4)dx =$

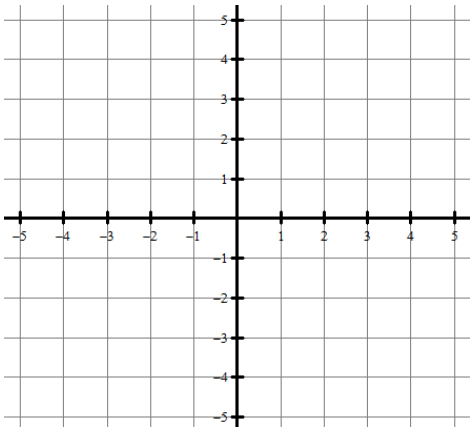


12. $\int_1^{-3} \left(-\frac{x}{2} + 1\right) dx =$

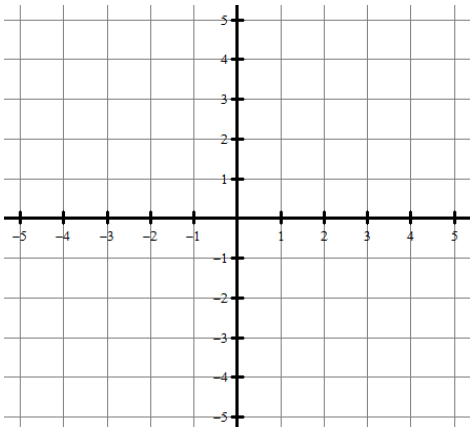


Sketch a graph of the definite integral without the calculator. Evaluate with the graphing calculator.

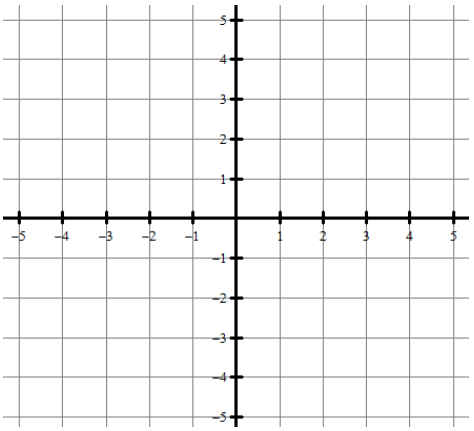
13. $\int_{-1}^{0} x^3\,dx =$



14. $\int_{-1}^{2}(1-x^2)dx =$



15. $\int_{-2}^{3}|x-2|dx =$



For 16-21 find each integral given *f* and *g* are continuous functions that

$\int_{-3}^{2} f(x)dx = 2$ $\int_{2}^{7} f(x)dx = -5$ $\int_{-3}^{2} g(x)dx = 6$

16. $\int_{2}^{7} 2f(x)dx$

17. $4\int_{-3}^{2} f(x)dx$

18. $\int_{-3}^{7} f(x)dx$

19. $\int_{-3}^{2} [g(x)-f(x)]dx$

20. $\int_{2}^{-3} g(x)dx$

21. $-\int_{7}^{2} f(x)dx$

For 22-27 find each integral given *f* and *g* are continuous functions that

$\int_{1}^{2} f(x)dx = -2$ $\int_{1}^{6} f(x)dx = 4$ $\int_{1}^{6} g(x)dx = 8$

22. $\int_{2}^{2} g(x)dx$

23. $\int_{6}^{1} g(x)dx$

24. $3\int_{1}^{2} f(x)dx$

25. $\int_{2}^{6} f(x)dx$

26. $\int_{1}^{6} [f(x)-g(x)]dx$

27. $\int_{1}^{6} [3f(x)-g(x)]dx$

Graph and answer the question using a graphing calculator.

28. For $0 \le t \le 8$, a particle is moving along the *x*-axis. The particle's position, $x(t)$, is not explicitly given. The velocity of the particle is given by $v(t) = e^{t/4}\cos\left(e^{t/4}\right)$ in meters per second.

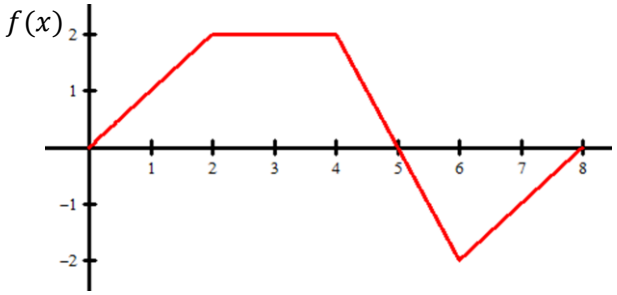
Find $\int_{0}^{8} v(t)dt$. What does this represent?

1. Suppose that the function f satisfies $f'(x) = 3x^2 - \sin \pi x$. Then the slope of the line tangent to the graph of f at the point $x = 2$.

   (A) 12
   (B) $8 - \frac{1}{\pi}$
   (C) 7
   (D) $12 - \pi$
   (E) 24

2. The graph of a piecewise linear function f, for $0 \le x \le 8$, is shown below. What is the value of $\int_0^8 f(x)dx$?

   (A) 1
   (B) 4
   (C) 8
   (D) 10
   (E) 13



3. The function f is given by $f(x) = x^4 + 4x^3$. On which of the following intervals is f decreasing?

   (A) $(-3,0)$
   (B) $(0,\infty)$
   (C) $(-3,\infty)$
   (D) $(-\infty,-3)$
   (E) $(-\infty,0)$

4. If $\int_2^5 f(x)dx = 12$ and $\int_5^8 f(x)dx = 4$, then all of the following must be true except

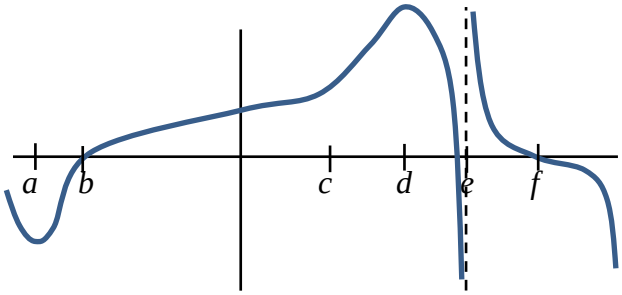
   (A) $\int_2^8 f(x)dx = 16$
   (B) $\int_2^5 f(x)dx - \int_5^8 3f(x)dx = 0$
   (C) $\int_5^2 f(x)dx = -12$
   (D) $\int_{-5}^{-8} f(x)dx = -4$
   (E) $\int_2^6 f(x)dx + \int_6^8 f(x)dx = 16$

5. $\frac{d}{dx}\tan^2(4x) =$

   (A) $8\tan(4x)$
   (B) $4\sec^2(4x)$
   (C) $8\tan(4x)\sec^2(4x)$
   (D) $4\tan(4x)\sec^2(4x)$
   (E) $8\sec^2(4x)$

6. Determine all the points on the graph below where the first derivative of the function is 0.

(A) a, b, e
(B) b, c, f
(C) a, d, e
(D) a, b, d, e
(E) a, d

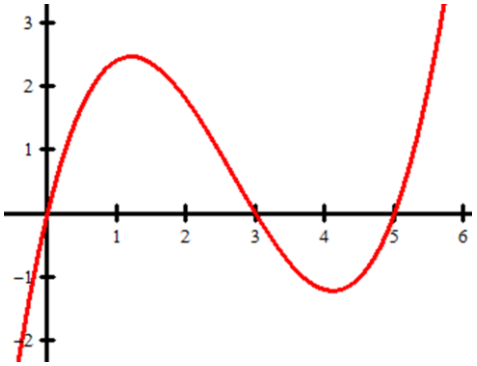


7. A 13-foot ladder is leaning against a 20-foot vertical wall when it begins to slide down the wall. During this sliding process, the bottom of the ladder is sliding away from the bottom of the wall at a rate of $\frac{1}{2}$ foot per second. Determine the rate at which the top of the ladder is sliding down the vertical wall when the tip of the ladder is exactly 5 feet above the ground.

(A) $-\frac{6}{5}$ feet per second
(B) $\frac{5}{6}$ feet per second
(C) $-\frac{12}{13}$ feet per second
(D) -2 feet per second
(E) Not enough information is given to solve this problem.

8. The graph of g is shown below. The area of the region between g and the x-axis on the interval $[0,3]$ is 9. The area of the region between g and the x-axis on the interval $[3,5]$ is 2. The value of $\int_0^5 g(x)dx$ is

(A) 5
(B) 7
(C) 9
(D) 11
(E) 18



CALCULATOR ACTIVE

9. What is the trapezoidal approximation of $\int_0^3 e^x dx$ using $n = 4$ subintervals?

(A) 6.407
(B) 13.565
(C) 19.972
(D) 27.879
(E) 34.944