Find the antiderivatives of the following.

1. $f'(x) = 9x^2 - 5x + 2$

$f(x) = \frac{9}{3}x^3 - \frac{5}{2}x^2 + 2x + C$

$f(x) = 3x^3 - \frac{5}{2}x^2 + 2x + C$

2. $f'(x) = \frac{x^4 - 4x^3 + 7x}{x}$

$f(x) = \frac{1}{4}x^4 - \frac{4}{3}x^3 + 7x + C$

3. $f'(x) = 2\sqrt{x} + 3$

$f'(x) = 2x^{1/2} + 3$

$f(x) = \frac{2}{3} \cdot 2x^{3/2} + 3x + C$

$f(x) = \frac{4}{3}x^{3/2} + 3x + C$

Evaluate the indefinite integrals.

4. $\int (3x + \pi)\, dx$

$\frac{3}{2}x^2 + \pi x + C$

5. $\int \left(x^{-3} + \frac{9}{x^2}\right) dx$

$\int (x^{-3} + 9x^{-2})\, dx$

$-\frac{1}{2}x^{-2} - 9x^{-1} + C$

$-\frac{1}{2x^2} - \frac{9}{x} + C$

6. $\int (5 - 6x^2)\, dx$

$5x - 2x^3 + C$

Evaluate the definite integrals using the Fundamental Theorem of Calculus.

7. $\int_0^4 (2x + 4)dx = x^2 + 4x\Big]_0^4 = 0$

$[(4)^2 + 4(4)] - [0^2 + 4(0)]$

$[16 + 16] - [0]$

$32 - 0$

32

8. $\int_{-1}^{3} (6x^2 - 8)dx = 2x^3 - 8x\Big]_{-1}^{3} = 24$

9. $\int_4^9 \sqrt{x}\, dx = \frac{2}{3}x^{3/2}\Big]_4^9 = \frac{38}{3}$

$\left[\frac{2}{3}\sqrt{9^3}\right] - \left[\frac{2}{3}\sqrt{4^3}\right]$

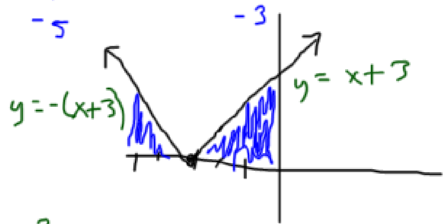
$\left[\frac{2}{3}(27)\right] - \left[\frac{2}{3}(8)\right]$

$\frac{54}{3} - \frac{16}{3}$

10. $\int_4^2 \left(\frac{x^2 - 1}{x^2}\right) dx = -\int_2^4 \left(1 - \frac{1}{x^2}\right) dx = -\left[x + \frac{1}{x}\right]_2^4 = \frac{-7}{4}$

Evaluate the definite integrals using the Fundamental Theorem of Calculus.

11. $\int_{-5}^{0} |x+3|\,dx = \int_{-5}^{-3} (-x-3)\,dx + \int_{-3}^{0} (x+3)\,dx = \frac{13}{2}$



$\left[-\frac{1}{2}x^2 - 3x\right]_{-5}^{-3} + \left[\frac{1}{2}x^2 + 3x\right]_{-3}^{0}$

$\left[-\frac{1}{2}(-3)^2 - 3(-3)\right] - \left[-\frac{1}{2}(-5)^2 - 3(-5)\right] + \left[\frac{1}{2}(0)^2 + 3(0)\right] - \left[\frac{1}{2}(-3)^2 + 3(-3)\right]$

$\left[-\frac{9}{2} + 9\right] - \left[-\frac{25}{2} + 15\right] \qquad + [0] - \left[-\frac{9}{2} + 9\right]$

$\frac{9}{2} - \frac{5}{2} \qquad + \frac{9}{2}$

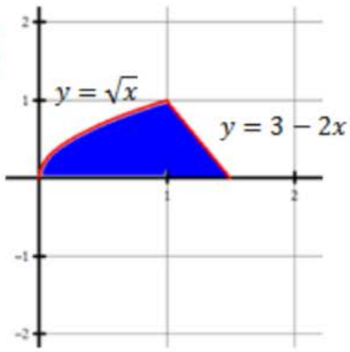
$\frac{4}{2} \qquad + \frac{9}{2} = \frac{13}{2}$

12. $\int_{-4}^{-1} \left(\frac{3}{x^2} + 1\right) dx = -\frac{3}{x} + x \Big]_{-4}^{-1} = \frac{21}{4}$

Find the area of the shaded region.

13.

$\frac{11}{12}$ units^2



$\int_0^1 \sqrt{x}\,dx + \int_1^{3/2} (3-2x)\,dx$

$\frac{2}{3}x^{3/2}\Big]_0^1 + 3x - x^2\Big]_1^{3/2}$

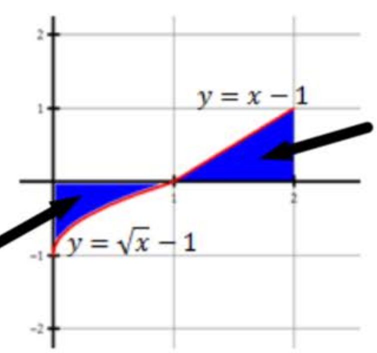
$\left[\frac{2}{3}\sqrt{1} - \frac{2}{3}\sqrt{0}\right] + \left(\left[3\left(\frac{3}{2}\right) - \left(\frac{3}{2}\right)^2\right] - \left[3(1) - (1)^2\right]\right)$

$\frac{2}{3} + \left(\left[\frac{9}{2} - \frac{9}{4}\right] - [3-1]\right)$

$\frac{2}{3} + \left(\frac{9}{4} - 2\right)$

$\frac{2}{3} + \frac{1}{4} = \frac{11}{12}$

14.

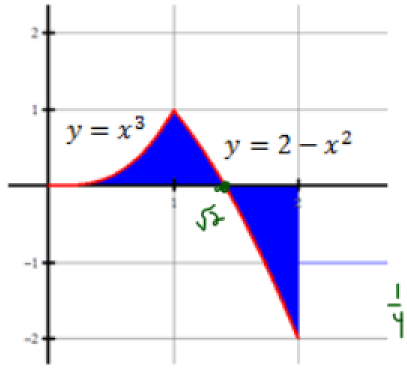


$\frac{1}{2}$

$\frac{1}{3}$

$\frac{1}{3} + \frac{1}{2} = \frac{5}{6}$ units^2

15.



$-\frac{11}{4} + \frac{8}{3}\sqrt{2}$ units^2

$$\int_0^1 x^3\,dx + \int_1^{\sqrt{2}} (2-x^2)\,dx + \int_{\sqrt{2}}^2 |2-x^2|\,dx$$

$$\frac{1}{4}x^4\Big]_0^1 \qquad 2x - \frac{1}{3}x^3\Big]_1^{\sqrt{2}} \qquad 2x - \frac{1}{3}x^3\Big]_{\sqrt{2}}^2$$

$$\frac{1}{4}(1)^4 - \frac{1}{4}(0)^4 \qquad \left[2\sqrt{2} - \frac{1}{3}(\sqrt{2})^3\right] - \left[2(1) - \frac{1}{3}(1)^3\right] \qquad \left[2(2) - \frac{1}{3}(2)^3\right] - \left[2\sqrt{2} - \frac{1}{3}(\sqrt{2})^3\right]$$

$$\frac{1}{4} \qquad \left[2\sqrt{2} - \frac{2}{3}\sqrt{2}\right] - \left[2 - \frac{1}{3}\right] \qquad \left[4 - \frac{8}{3}\right] - \left[2\sqrt{2} - \frac{2}{3}\sqrt{2}\right]$$

$$\frac{4}{3}\sqrt{2} - \frac{5}{3} \qquad \left|\frac{4}{3} - \frac{4}{3}\sqrt{2}\right|$$

$$\frac{1}{4} + \frac{4}{3}\sqrt{2} - \frac{5}{3} - \left(\frac{4}{3} - \frac{4}{3}\sqrt{2}\right)$$

$$\frac{1}{4} + \frac{4}{3}\sqrt{2} - \frac{5}{3} - \frac{4}{3} + \frac{4}{3}\sqrt{2}$$

$$\frac{1}{4} - \frac{9}{3} + \frac{8}{3}\sqrt{2}$$

$$\frac{1}{4} - 3 + \frac{8}{3}\sqrt{2}$$

$$-\frac{11}{4} + \frac{8}{3}\sqrt{2}$$

8.2 The 1st Fundamental Theorem of Calculus

TEST PREP

MULITPLE CHOICE

1. **A**
2. **D**
3. **E**
4. **D**

FREE RESPONSE

Your score:___ out of 7

1. A particle moves along the y-axis with velocity $v(t) = -\frac{2}{\pi}\sin\left(\frac{\pi}{2}t\right)$ cm/sec for $t \geq 0$ in seconds.

(A) In what direction is the particle moving at $t = \frac{1}{3}$? Justify.

$v\left(\frac{1}{3}\right) = \frac{-1}{\pi}$

1 point

The particle is moving down because the velocity at $t = \frac{1}{3}$ is negative.

(B) Find the earliest time, $t_1 > 0$, when the particle changes direction.

$v(t) = 0$

$-\frac{2}{\pi}\sin\left(\frac{\pi}{2}t\right) = 0$

$t = 2, 4, 6, 8, 10\ldots$

$t = 2n$, n is an integer

1 point

First Derivative Test

t	$\frac{1}{2}$	2	$\frac{5}{2}$
$v(t)$	$v(\frac{1}{2}) < 0$	0	$v(\frac{5}{2}) > 0$
	neg	min	pos

check the critical point t=2. Since the derivative is decreasing and the increasing, it is a relative min which means the particle changes direction

1 point

OR

Second Derivative Test

$v'(2) = 1$

Since the second derivative is positive, $t = 2$ is a relative min which means the particle changes direction.

(C) What is the particle's average acceleration over the interval $[0, t_1]$?

$$\frac{v(2) - v(0)}{2 - 0} = 0$$

1 point 1 point

(D) Does the concavity of the position function, $s(t)$, change sign over the interval $[0, t_1]$?

$v'(t) = a(t) = 0$

$-\cos\left(\frac{\pi}{2}t\right) = 0$

$t = 1, 3, 5, 7, 9\ldots$

$t = 1 + 2n$, n is an integer

1 point

interval	$0 < t < 1$	1	$1 < t < 2$
test	$\frac{1}{2}$	1	$\frac{3}{2}$
$v'(t)$	$v'(\frac{1}{2}) = -\frac{\sqrt{2}}{2}$	0	$v'(\frac{3}{2}) = \frac{\sqrt{2}}{2}$
conclusion	negative concave down	pos	positive concave up

1 point

The concavity of the position function changes sign at $t = 1$ from concave down to concave up